

Python, III deo: PyLab i SymPy

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Šta je PyLab?

- ▶ environment koji cine
 - ▶ NumPy
 - ▶ SciPy
 - ▶ Matplotlib
 - ▶ IPython
- ▶ kako se instalira pylab?
- ▶ Ubuntu: Software Center ili Synaptic
- ▶ win: <http://www.enthought.com/>
- ▶ ... Canopy

IDE? Spyder!

- ▶ potrebno?
- ▶ kako kome, kako za šta ...
- ▶ IPython meni sasvim dobar ...
- ▶ ... ima i Qt verziju, IPython Qt console
- ▶ ... mada postoji i Spyder
- ▶ <https://pypi.python.org/pypi/spyder>
- ▶ ako nema dovucite iz repository ...
- ▶ ima i pod win, ali ...

NumPy

- ▶ <http://www.numpy.org/>
- ▶ osnovna biblioteka za numerički zahtevne Python primene, sadrži:
 1. N-dimensional array object
 2. array slicing methods
 3. array reshaping methods
- ▶ i module za:
 1. basic linear algebra functions
 2. basic Fourier transforms
 3. advanced random number capabilities
- ▶ pokrenete IDLE
- ▶ `import numpy`
- ▶ `dir(numpy)`
- ▶ `help(numpy)`
- ▶ `del numpy`

NumPy

- ▶ obradićemo, sve, naravno, imamo vremena, ...
- ▶ evolutivno nastao, „haotično“ iz Numeric i NumArray
- ▶ znanje koje raste
- ▶ Travis E. Oliphant & Enthought, biće još kontakata sa njima
- ▶ <http://www.enthought.com/>
- ▶ nema načina da se zapamti i nije „zauvek“
- ▶ potreban nov način učenja i snalaženja
- ▶ izbeći reinventing
- ▶ masovno korišćenje help-a i primera
- ▶ Matlab-Python-R

SciPy

- ▶ <http://www.scipy.org/>
- ▶ scientific lib za Python, zavisi od NumPy
- ▶ nekoliko modula in a single package, kao i NumPy
- ▶ moduli za:
 1. statistics
 2. optimization
 3. numerical integration
 4. linear algebra
 5. Fourier transforms
 6. signal processing
 7. image processing
 8. ODE solvers
 9. special functions

SciPy

- ▶ i dalje u IDLE
- ▶ `import scipy`
- ▶ `dir(scipy)`
- ▶ `help(scipy)`
- ▶ `del scipy`
- ▶ dobro razmislite pre nego što krenete u reinvent!
- ▶ ja ovo ne mogu da zapamtim, sto zapamtim zaboravim
- ▶ pomenuh li nov način učenja?
- ▶ kako organizovati informacije, previše ih je (i kratko traju)
- ▶ “Numerical Recipes”

matplotlib

- ▶ <http://matplotlib.sourceforge.net/>, sjajan sajt
- ▶ package sa dugim nizom modula
- ▶ jako dobro se vidi strukturiran namespace, package.modul
- ▶ Python 2D plotting library (samo 2D?)
- ▶ ono što ima sjajno je, a sada ima skoro sve
- ▶ galerija i primeri
- ▶ uputstvo, 3.0.0, pdf, 2288 strana, 21.09.2018.
- ▶ John D. Hunter, video lectures
- ▶ sintaksa vrlo liči na Matlab i Octave
- ▶ skoro kao gnuplot, ali direktno, bez izlaženja u gnuplot

IPython

- ▶ <http://ipython.org/>
- ▶ <https://jupyter.org/>
- ▶ interaktivni Python environment, vrlo nalik na Octave, wxMaxima, ...
- ▶ autocompletion by tab
- ▶ doteruje komande da budu shvaćene
- ▶ od mnogo mogućnosti: store, history, logging, ...

PyLab

- ▶ sve prethodno zgodno spakovano
- ▶ sređen namespace da ne mora puno dot notation
- ▶ sintaksa jako liči na Octave/Matlab
- ▶ počinjemo, komandna linija, terminal
- ▶ `ipython --pylab`

PyLab, IPython, osnovno

```
help()
?  
%quickref  
a = 3  
b = 'string'  
lista = [1, 2, 3]  
li<tab>  
whos
```

PyLab, IPython, store

```
store a
store b
store lista
del a
del b
a
b
store -r
a
b
store -z
store
```

Pylab, matematika, konstante

```
pi
e
j
1j
exp(1j * pi)
math.exp(1j * pi)
help(exp)
help(math.exp)
exp?
math.exp?
e**(1j * pi) + 1
```

PyLab, matrice 1

```
a = array([[1, 2], [3, 4]])
```

```
a
```

```
a.size
```

```
a.shape
```

```
a.ndim
```

```
a.dtype
```

```
a.dtype.name
```

```
a.itemsize
```

```
a.transpose()
```

```
transpose(a)
```

```
a.T
```

Pylab, matrice 2

```
det(a)
eig(a)
b = eig(a)
type(b)
len(b)
type(b[0])
type(b[1])
c, d = b
c.size
c.shape
c.ndim
c.dtype.name
c.itemsize
```

Pylab, matrice 3, inverzija i množenje

```
aa = inv(a)
aa
aa.dtype.name
aa * a
a * aa
dot(a, aa)
dot(aa, a)
x = arange(10)
x
print x
dot(x, x)
```

Sve operacije su **elementwise!!!**
Velika razlika u odnosu na **Octave!!!**

PyLab, gde je dot, tu je cross

```
i = array([1, 0, 0])
j = array([0, 1, 0])
print dot(i, j), dot(j, i)
print cross(i, j)
print cross(j, i)
cross?
source(cross)
```

Poznato od nekud? Rekoh da već ima, ...

Teško je naći potpuno nov primer ...

Reinventing problem! Google pomaže puno!

Pylab, rang!!!

```
a = array([[1, 1], [1, 1]])  
rank(a)    # ???  
help(rank) # sad jasno?  
linalg.matrix_rank(a)
```

Pazite!!!

Nažalost, lično iskustvo, ne tuđe!

PyLab, inicijalizacija, neke posebne matrice

```
I = eye(3)
print I
nule = zeros(4)
print nule
zeros?
zeros((4, 2))
zeros(4, 2)    # pazite!
zeros((4, 2), dtype = int)
zeros((4, 2), dtype = complex)
ones([3, 4])
empty([6, 4])
empty?
```

PyLab, još elementwise operacija

`a + 1`

`a * a`

`a**3`

`a / 2`

`a / 2.`

Pazite na /, velika razlika u odnosu na Octave!!!

Pylab, reshape

```
help(reshape)
print a
a.reshape(1, 4)
a.reshape(4, 1)
reshape(a, (1, 4))
reshape(a, (4, 1))
a.reshape(2, 3)
a.reshape(4)
a.reshape((4, ))
reshape(a, 4)
reshape(a, (4, ))
```

PyLab, in place array operations

```
xx = arange(20)
print xx
xx += 1
print xx
xx = xx.reshape(4, 5)
print xx
xx -= 1
print xx
xx *= 2
print xx
```

PyLab, transpose, revisited

```
help(transpose)
xx = arange(5) + 1
print xx
xx.shape
xx.ndim
xx = xx.transpose()
xx.shape
print xx
xx = xx.reshape(1, 5)
xx.ndim
print xx
xx.shape
xx = xx.transpose()
xx.shape
print xx
```

Pylab, eig, revisited

```
a = eye(2)
print a
b = eig(a)
type(b)
len(b)
c, d = b
print c
print d
a[0, 1] = 1
print a
b = eig(a)
c, d = b
print c
print d
```


Pylab, plot 1

```
x = arange(10) + 0.5 # nikako 1/2
y = x**2
plot(x, y)
plot(x, y, 'r+')
xlabel('x')
ylabel('y')
title('y = x**2')
```

pretty matlabeće, ali nema `hold on`
save kako hocete, odaberete format
u IDLE za ovo je bilo potrebno `ion()` (zaglavljivanje!)
pogledati `show()` i `ion()` help

`close()` ili `close('all')`

Pylab, plot 2

```
close('all')
phi = linspace(0, 2 * pi, 1000)
x = 2 * cos(phi)
y = 2 * sin(phi)
plot(x, y)
axis([-3, 3, -3, 3])
axis('equal')
axis([-3, 3, -3, 3])
axis([-4, 4, -3, 3])
title('kruznica')
```

```
axis?
```

Pylab, plot 3

```
close('all')  
  
help.figure)  
  
figure(figsize = (6, 6))  
plot(x, y)  
axis('equal')  
axis([-3, 3, -3, 3])  
title('kruznica')
```

Pazite na redosled 'equal' i [-3, 3, -3, 3]!!!

Pylab, plot 4, histograms

```
x = rand(10000)
plot(x)
close()
rand?
```

```
hist(x, 100)
close()
```

```
hist?
```

Pylab, plot 5, normal distribution

```
x = randn(10000)
plot(x)
close()
```

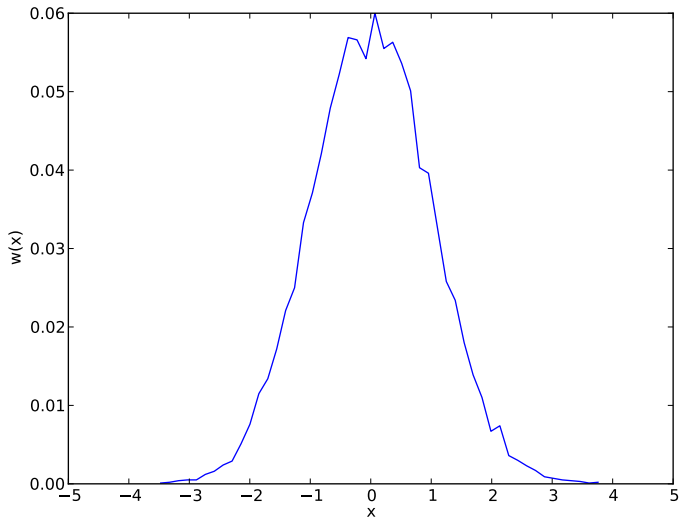
```
randn?
```

```
hist(x, 100)
close()
```

```
t = hist(x, 50)
type(t)
len(t)
len(t[0])
len(t[1])
```

Pylab, plot 6

```
y = t[0] / 10000.0    # pazite kod !  
x = t[1]  
len(y)  
len(x)  
xx = (x[0 : len(x)-1] + x[1 : len(x)]) / 2  
len(xx)  
close()  
plot(xx, y)  
xlabel('x')  
ylabel('w(x)')  
xlim(-5, 5)  
xticks(linspace(-5, 5, 11))  
sum(y)  
help(savefig)  
savefig('slika') # potrazite slika.png  
savefig('slika.pdf') # potrazite slika.pdf
```



Pylab, jedan script, dat.py

```
from pylab import *

deg = linspace(0, 4*360, 4*360 + 1)
wt = radians(deg)

f = exp(- wt / 2 / pi * 0.5) * sin(wt)
fp = exp(- wt / 2 / pi * 0.5)
fm = -fp

dat = array([deg, wt, f, fp, fm]).transpose()

np.save('deg.npy', deg)
np.save('f.npy', f)
np.save('fp.npy', fp)

np.savetxt('dat.txt', dat, fmt='%.4f')
```


Pylab, run, #1

na komandnoj liniji:

```
python dat.py  
more dat.txt  
less dat.txt  
ls *.npy  
ipython --pylab
```

Pylab, run, #2 i #3

inside IPython:

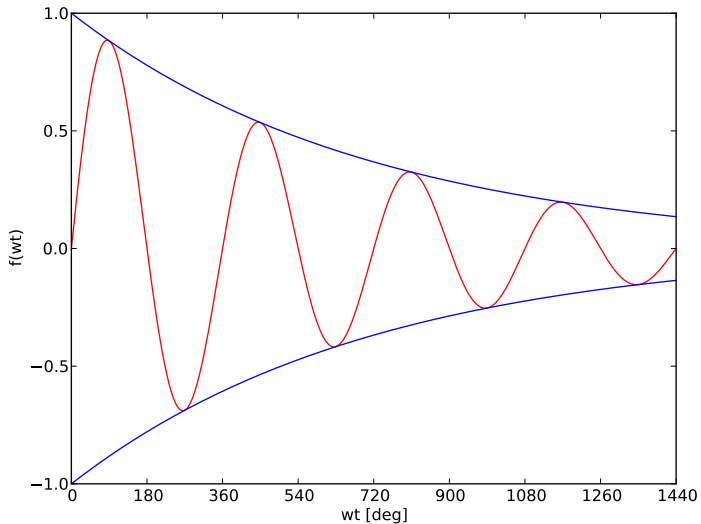
```
run dat.py  
execfile('dat.py')
```

execfile radi i kod IDLE

PyLab, after the run

```
np.save?  
np.savetxt?  
np.load?  
deg = np.load('deg.npy')  
f = np.load('f.npy')  
fp = np.load('fp.npy')  
fm = -fp  
plot(deg, f, 'r')  
plot(deg, fp, 'b')  
plot(deg, fm, 'b')  
xlim(0, 360 * 4)  
xticks(arange(0, 360 * 4 + 1, 180))  
xlabel('wt [deg]')  
ylabel('f(wt)')  
savefig('datslik')  
close()
```

datslik.pdf

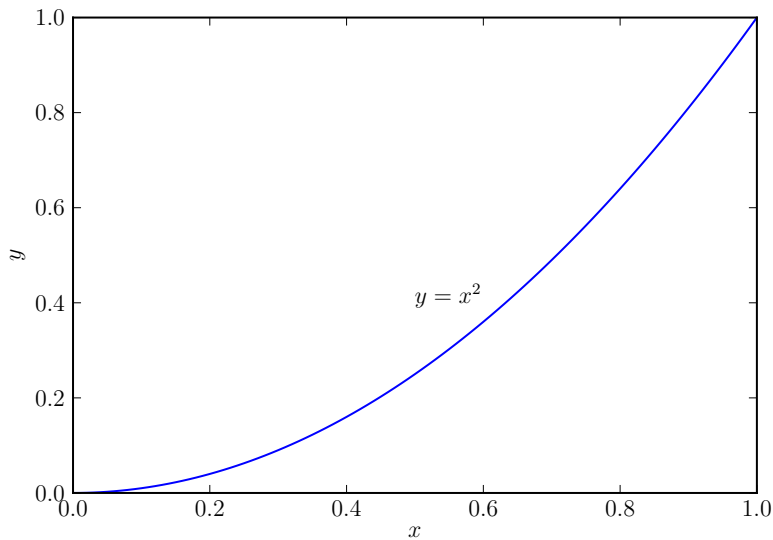


PyLab, L^AT_EX, estetika

```
help(rc)

close('all')
x = linspace(0, 1, 101)
y = x**2
rc('text', usetex = True)
rc('font', family = 'serif')
figure(0, figsize = (6, 4))
plot(x, y)
xlabel(r'$x$')
ylabel(r'$y$')
text(0.5, 0.4, r'$y = x^2$')
savefig('kvadrat.pdf', bbox_inches = 'tight')
```

kvadrat.pdf



Pylab, L^AT_EX, „standardna“ „preambula“

```
rc('text', usetex = True)
rc('font', family = 'serif')
rc('font', size = 12)
rcParams['text.latex.preamble']=[r'\usepackage{amsmath}']
```

PyLab, linear algebra, 0th part

hoću da rešim sistem jednačina:

$$x + y = 3$$

$$x - y = 1$$

```
a = array([[1, 1], [1, -1]])
```

```
print a
```

```
b = array([[3], [1]])
```

```
print b
```

```
x = solve(a, b)
```

```
print x
```

```
b = array([3, 1])
```

```
print b
```

```
x = solve(a, b)
```

```
print x
```


PyLab, linear algebra, 1st part

```
randn?
```

```
linalg.lstsq?
```

```
x = linspace(0, 2, 201)
```

```
y = x / 2
```

```
xe = linspace(0, 2, 21)
```

```
ye = xe / 2 + randn(21) * 0.1
```

```
A = array([xe, ones(len(xe))]).transpose()
```

```
t = linalg.lstsq(A, ye)
```

```
print t
```

```
type(t)
```

```
len(t)
```

```
a = t[0][0]
```

```
b = t[0][1]
```

Pylab, linear algebra, 2nd part

```
close('all')
```

```
rc('text', usetex=True)
```

```
rc('font', family='serif', size='16')
```

```
plot(x, y, 'b')
```

```
plot(xe, ye, 'r.')
```

```
yfit = a * x + b
```

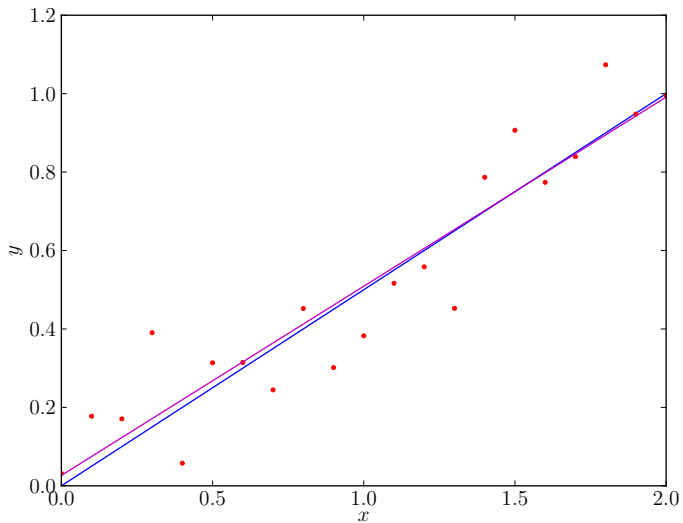
```
plot(x, yfit, 'm')
```

```
xlabel(r'$x$')
```

```
ylabel(r'$y$')
```

```
savefig('fitovanje.pdf')
```

fitovanje.pdf



Šta je SymPy?

- ▶ paket za simboličko računanje koristeći Python
- ▶ cilj: isto što i wxMaxima
- ▶ ideja: Python sintaksa, poznata
- ▶ moduli, funkcije za simboličko računanje
- ▶ <http://sympy.org/en/index.html>
- ▶ <https://github.com/sympy/sympy/releases>
- ▶ <http://live.sympy.org/>
- ▶ uputstvo 1.3, 2044 strane, 14.09.2018; čitate stranu po stranu?
- ▶ Ubuntu: Software Center ili Synaptic
- ▶ volite Mathematica sintaksu?
- ▶ **odličan pregled i tutorial**
- ▶ za one koji vole matematiku <http://www.sagemath.org/>

SymPy, počinjemo

- ▶ komandna linija, `isympy`
- ▶ stari poznanik, IPython, customized again
- ▶ pogledajte šta radi na početku, deklaracije `symbols` ...

SymPy, polinomi

```
p1 = x**2 + 2 * x + 1
p1
print p1
pprint(p1)
p2 = x + 1
p2
p3 = x**2 - 4
p3
p1 * p2 * p3
_ .expand()
```

SymPy, expand i apart

```
p1 / p2
_.expand()
p1 / p2
_.apart()
apart(p1 / p2, x)
help(expand)
help(apart)
f = 1 / (x**2 * (x + 1))
apart(f, x)
together(_, x)
_.expand()
_.factor()
_.subs(x, 3)
```

SymPy, opet problemi sa deljenjem

1 / 3

Rational(1, 3)

S('1/3')

S('1 / 2')

S('1/2') - S('1/3')

SymPy, konstante i izračunavanje

```
p2 = pi**2
p2
p2.evalf()
p2.evalf(100)
p2.n()
p2.n(200)
N(p2)
N(p2, 300)
exp(1)
e
E
E.evalf()
I**2
exp(I * pi) + 1
E**(I * pi) + 1
```

SymPy, Ramanujan

```
r = E**(pi * sqrt(163))  
r  
r.n(20)  
r.n(21)  
r.n(22)  
r.n(23)  
r.n(24)  
r.n(25)  
r.n(28)  
r.n(30)  
r.n(35)  
r.n(500)
```

SymPy, realni brojevi

```
exp(I * x).expand(complex = True)
xr = Symbol('xr', real = True)
exp(I * xr).expand(complex = True)
```

SymPy, linearna algebra

$$e1 = 2 * x + y - 4$$

$$e2 = x - y + 1$$

e1

e2

```
solve((e1, e2), (x, y))
```

$$e3 = x - y + 4$$

e2

e3

```
solve((e2, e3), (x, y))
```

$$e4 = 3 * x - 3 * y + 12$$

e3

e4

```
solve((e3, e4), (x, y))
```

SymPy, linearna algebra, opet isto ...

```
e1 = Eq(2 * x + y, 4)
```

```
e1
```

```
e2 = Eq(x - y, -1)
```

```
e1
```

```
e2
```

```
solve((e1, e2), (x, y))
```

```
e3 = Eq(x - y, -4)
```

```
e2
```

```
e3
```

```
solve((e2, e3), (x, y))
```

```
e4 = Eq(3 * x - 3 * y, -12)
```

```
e3
```

```
e4
```

```
solve((e3, e4), (x, y))
```

SymPy, matrice, samo osnovno

```
A = Matrix([[x, 1], [1, y]])
A
print A
pprint(A)
A**2
A**(-1)
B = A.inv()
B
A**(-1) - B
A * B
simplify(A * B)
```

SymPy, ne baš linearna algebra

```
solve(p1, x)
solve(p2, x)
solve(p3, x)
solve(x**4 - 1, x)
solve(Eq(x**4, 1), x)
```

SymPy, limesi

```
f = sin(14 * x) / x
g = 1 / x
limit(f, x, 0)
limit(g, x, 0)
limit(g, x, 0, dir = '+')
limit(g, x, 0, dir = '-')
limit?
```


SymPy, izvodi

```
print p1
pprint(p1)
diff(p1, x)
diff(p1, x, 2)
diff(p1 * p2 * p3, x)
_.expand()
diff(p1 * p2 * p3, x, 14)
diff(p1*exp(3*x), x)
_.expand()
```

SymPy, integrali

```
integrate(p1, x)  
integrate(p1, (x, 1, 2))  
integrate(cos(x), x)  
integrate(sin(x), (x, 0, pi))
```

SymPy, Taylor

```
sin(x).series(x, 0, 10)
series(sin(x), x, 0, 10)
series?
series(sin(x), x, 1, 10)
series(exp(x), x, 0, 5)
series(exp(x), x, 5, 5)
source(series)
```

SymPy, da obrišemo funkciju

```
f  
del f  
f = Function('f')  
f
```

SymPy, diferencijalne jednačine

trigonometrijske funkcije kao rešenje

```
f(t).diff(t, 2) + 4 * f(t)
```

```
dsolve(_, f(t))
```

```
f(t).diff(t, 2) + 4 * f(t) - 24 * cos(4*t)
```

```
dsolve(_, f(t))
```

i eksponencijalne ...

```
f(x).diff(x, 2) - 4 * f(x)
```

```
dsolve(_, f(x))
```

```
Eq(f(x).diff(x, 2) - 4 * f(x), - 4 * exp(- 2 * x))
```

```
dsolve(_, f(x))
```

SymPy, diferencijalne jednačine, provera

```
deq = Eq(f(t).diff(t, 2) - 4 * f(t), - 4 * exp(-2*t))
deq
deq.lhs
deq.rhs
sol = dsolve(deq, f(t))
sol
sol.lhs
sol.rhs
s = sol.rhs
s
ver = deq.subs(f(t), s)
ver
ver = ver.doit()
ver
ver.lhs.expand()
ver.rhs
ver.expand()
```

SymPy, diferencijalne jednačine, script, problem

Rešiti diferencijalnu jednačinu

$$\frac{d^2 f(t)}{dt^2} + 2 \frac{d f(t)}{dt} + 4 f(t) = 2 \sin(t)$$

i proveriti rešenje.

Napisati Python script koji rešava problem, prikazati jednačinu, rešenje i rezultat provere.

SymPy, diferencijalne jednačine, script

```
from __future__ import division
from sympy import *

# cast
y, t = symbols('y t')
f = symbols('f', cls=Function)

# equation
deq = Eq(f(t).diff(t, 2) + 2 * f(t).diff(t, 1) + 4 * f(t), 2 * sin(t))
print
pprint(deq)

# solution
sol = dsolve(deq, f(t))
print
pprint(sol)

# verification
s = sol.rhs
ver = deq.subs(f(t), s)
ver = ver.doit()
ver = ver.expand()
print
pprint(ver)
```


SymPy, diferencialne jednačine, script, rešenje

$$4f(t) + 2\frac{d}{dt}(f(t)) + \frac{d^2}{dt^2}(f(t)) = 2\sin(t)$$

$$f(t) = \frac{1}{\sqrt{3}}C_1\sin\left(\frac{t}{\sqrt{3}}\right) + C_2\cos\left(\frac{t}{\sqrt{3}}\right)e^{-t} + \frac{6\sin(t)}{13} - \frac{4\cos(t)}{13}$$

True

SymPy: :, =, is()

- ▶ na kraju, a moglo je i na početku ...

Maxima	vs.	SymPy	objašnjenje
:	je	=	dodela vrednosti
is()	je	==	logički operator
=	je	Eq(*, *)	deklarisanje jednakosti

- ▶ očigledno postoji potreba za različitim =

in SymPy:

```
x**4 = 1      # error
x**4 == 1     # poređenje
Eq(x**4, 1)   # jednakost, jednačina
solve(_, x)   # koja može da se reši
```

PSAE, šta nismo uradili, a trebalo je, t ≪ (potrebnog)

- ▶ **komandna linija**, <http://linuxcommand.org/tlcl.php>
- ▶ **regular expressions**, Charles Severance, video, 35', 23"
- ▶ detaljnije Code::Blocks ili neki drugi IDE
- ▶ **Eclipse** <http://www.eclipse.org/>
- ▶ ukratko LibreOffice, više pravila pisanja
- ▶ malo više vremena za Octave, ipak je jako dobar program
- ▶ makar malo vremena za **SciLab**, Xcos pre svega
- ▶ malo vremena za **GIMP**, mada nije problem
- ▶ malo vremena za **Inkscape**, ovo je veći problem
- ▶ **Qucs** <http://qucs.sourceforge.net/>
- ▶ **Ngspice** <http://ngspice.sourceforge.net/>
- ▶ moj izbor: **julia** i **sage**
- ▶ ovo je samo početak, mada je za sada ...

— K R A J —