

SINTEZA PASIVNIH FILTARA

REALIZACIJA ULAZNIH FUNKCIJA LC MREŽE

$$F_{LC}(s) \in \{Z_{LC}(s), Y_{LC}(s)\}$$

1. Svi polovi i nule $F_{LC}(s)$ su prosti i leže na imaginarnoj osi u s ravni.
2. Polovi i nule $F_{LC}(s)$ su naizmenično poredani na $j\omega$ osi.
3. Tačke $s = 0$ i $s \rightarrow \infty$ su ili pol ili nula $F_{LC}(s)$.
4. $F_{LC}(s)$ je neparna racionalna funkcija čiji se stepeni u brojiocu i imeniocu razlikuju tačno za 1.
5. Svi ostaci $F_{LC}(s)$ u polovima su realni i pozitivni:

Za prost pol:

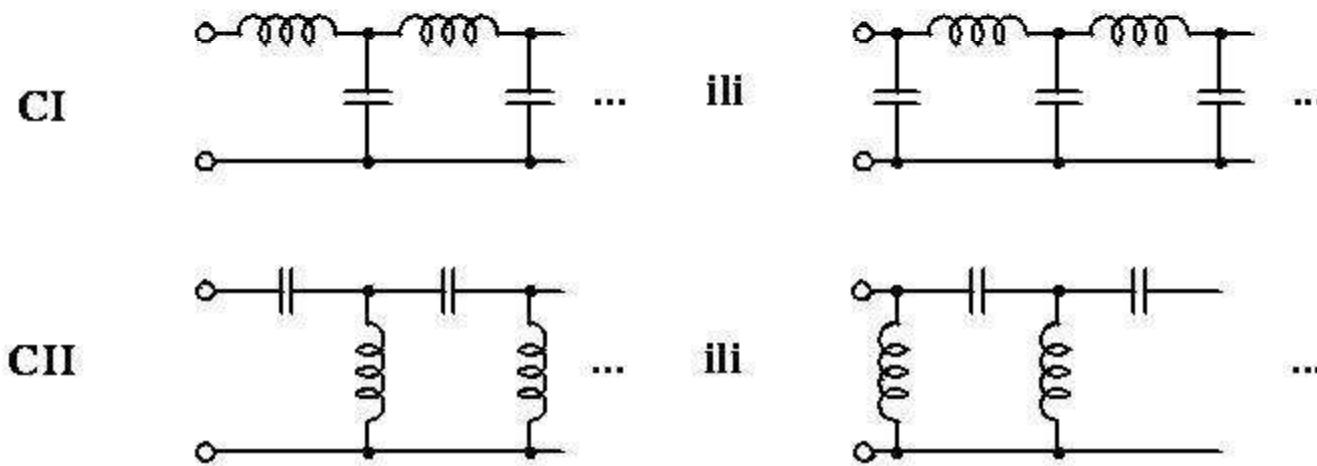
$$\alpha_k = \left[(s - s_k) F_{LC}(s) \right]_{s=s_k}, \text{ za } s_k \text{ konačno}$$

$$\alpha_k = \lim_{s \rightarrow \infty} \left[\frac{1}{s} F_{LC}(s) \right], \text{ za } s_k \rightarrow \infty$$

Postoje različiti postupci realizacije, a najčešće se koriste Kauerove (Cauer) forme

- **Kauerov I oblik (CI) za $F_{LC}(s)$ čiji je brojilac za 1 većeg reda od imenioca**
- **Kauerov II oblik (CII) za $F_{LC}(s)$ čiji je imenilac neparni polinom**

Kola koja realizuju Kauerove forme imaju lestvičastu strukturu

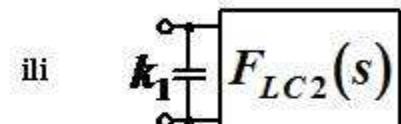
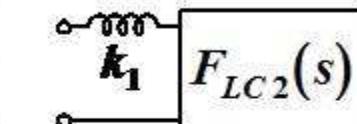


Kauerova I forma

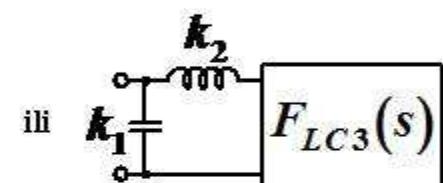
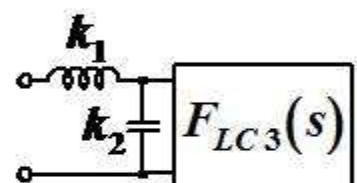
Svaki element se dobija izdvajanjem pola u beskonačnosti

$$F_{LC}(s) = \frac{A(s)}{B(s)} \quad p(s) = A(s) + B(s) \text{ je Hurvicov polinom}$$

$$F_{LC}(s) = F_{LC1}(s) = k_1 s + F_{LC2}^{-1}(s) \quad k_1 = \lim_{s \rightarrow \infty} \left[\frac{1}{s} F_{LC1}(s) \right]$$

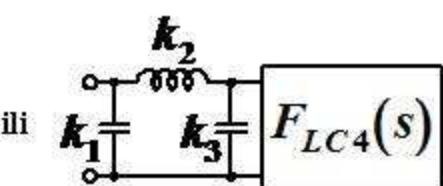
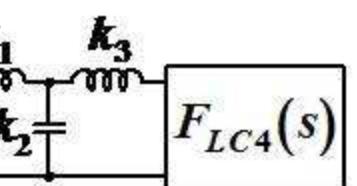


$$F_{LC2}^{-1}(s) = F_{LC1}(s) - k_1 s$$



$$F_{LC3}^{-1}(s) = F_{LC2}(s) - k_2 s$$

$$F_{LC3}(s) = k_3 s + F_{LC4}^{-1}(s) \quad k_3 = \lim_{s \rightarrow \infty} \left[\frac{1}{s} F_{LC3}(s) \right]$$



⋮

$$F_{LC}(s) = k_1 s + \frac{1}{k_2 s + \frac{1}{k_3 s + \frac{1}{\vdots + \frac{1}{k_{n-1} s + \frac{1}{k_n s}}}}}, \text{ gde je } n \text{ red polinoma u brojiocu } F_{LC}(s)$$

Primer: Realizovati u Kauerovoj I formi $Z(s) = \frac{s^4 + 10s^2 + 9}{s^5 + 20s^3 + 64s}$

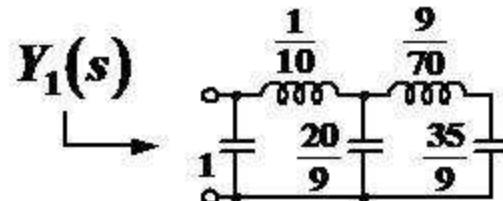
Može da se realizuje $\frac{1}{Z(s)}$

$$Y_1(s) = \frac{s^5 + 20s^3 + 64s}{s^4 + 10s^2 + 9} = 1s + \frac{10s^3 + 55s}{s^4 + 10s^2 + 9} \quad k_1 = \lim_{s \rightarrow \infty} \frac{Y_1(s)}{s} = 1$$

$$Z_2(s) = \frac{s^4 + 10s^2 + 9}{10s^3 + 55s} = \frac{1}{10}s + \frac{\frac{9}{2}s^2 + 9}{10s^3 + 55s} \quad k_2 = \lim_{s \rightarrow \infty} \frac{Z_2(s)}{s} = \frac{1}{10}$$

$$Y_3(s) = \frac{10s^3 + 55s}{\frac{9}{2}s^2 + 9} = \frac{20}{9}s + \frac{35s}{\frac{9}{2}s^2 + 9} \quad k_3 = \lim_{s \rightarrow \infty} \frac{Y_3(s)}{s} = \frac{20}{9}$$

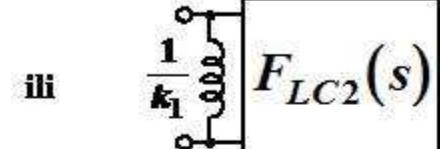
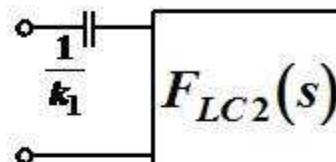
$$Z_4(s) = \frac{\frac{9}{2}s^2 + 9}{35s} = \frac{9}{70}s + \frac{1}{\frac{35}{9}s} \quad Y_1(s) = 1s + \frac{1}{\frac{1}{10}s + \frac{1}{\frac{20}{9}s + \frac{1}{\frac{9}{70}s + \frac{1}{\frac{35}{9}s}}}}$$



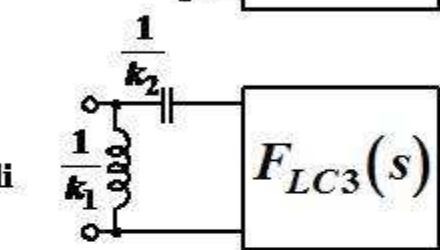
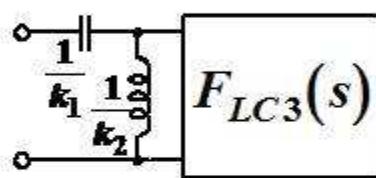
Kauerova II forma

$F_{LC}(s) = \frac{\text{paran polinom}}{\text{neparan polinom}} \Rightarrow \text{postoji pol u } s=0 \rightarrow \text{on se izdvaja iz funkcije}$

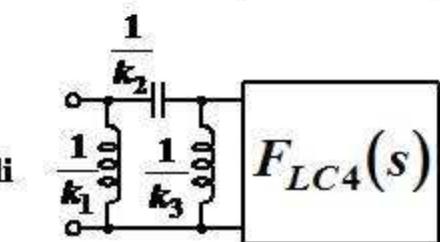
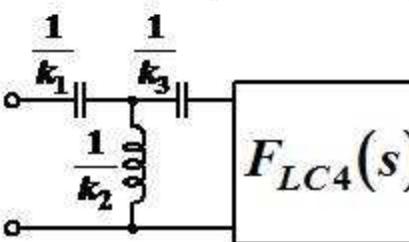
$$F_{LC}(s) = F_{LC1}(s) + \frac{k_1}{s} + F_{LC2}^{-1}(s) \quad k_1 = sF_{LC1}(s)|_{s \rightarrow 0}$$



$$F_{LC2}^{-1}(s) = F_{LC1}(s) - \frac{k_1}{s}$$



$$F_{LC2}(s) = \frac{k_2}{s} + F_{LC3}^{-1}(s) \quad k_2 = sF_{LC2}(s)|_{s \rightarrow 0}$$



$$F_{LC3}^{-1}(s) = F_{LC2}(s) - \frac{k_2}{s}$$

$$F_{LC3}(s) = \frac{k_3}{s} + F_{LC4}^{-1}(s) \quad k_3 = sF_{LC3}(s)|_{s \rightarrow 0}$$

⋮

$$F_{LC}(s) = \frac{k_1}{s} + \frac{1}{\frac{k_2}{s} + \frac{1}{\frac{k_3}{s} + \frac{1}{\dots + \frac{1}{\frac{k_{n-1}}{s} + \frac{1}{k_n}}}}}, \text{ gde je } n \text{ red polinoma u imeniku } F_{LC}(s)$$

Primer: Realizovati u Kauerovoj II formi $Z(s) = \frac{s^4 + 10s^2 + 9}{s^5 + 20s^3 + 64s}$

Prvi način: $Z_1(s) = \frac{9 + 10s^2 + s^4}{64s + 20s^3 + s^5} = \frac{9}{64s} + Y_2^{-1}(s); \quad Y_2^{-1}(s) = Z_1(s) - \frac{9}{64s}; \dots$

Drugi način: sменом $p = \frac{1}{s}$ своди се на развијање Kauerove I forme

$$Z(p) = \frac{\frac{1}{p^4} + \frac{10}{p^2} + 9}{\frac{1}{p^5} + \frac{20}{p^3} + \frac{64}{p}} = \frac{p + 10p^3 + 9p^5}{1 + 20p^2 + 64p^4}$$

$$Z_1(p) = \frac{9}{64}p + \frac{\frac{460}{64}p^3 + \frac{55}{64}p}{64p^4 + 20p^2 + 1} \quad k_1 = \frac{9}{64}$$

$$Y_2(p) = \frac{64p^4 + 20p^2 + 1}{\frac{115}{16}p^3 + \frac{55}{64}p} = \frac{1024}{115}p + \frac{\frac{284}{23}p^2 + 1}{\frac{115}{16}p^3 + \frac{55}{64}p} \quad k_2 = \frac{1024}{115}$$

$$Z_3(p) = \frac{\frac{115}{16}p^3 + \frac{55}{64}p}{\frac{284}{23}p^2 + 1} = \frac{2645}{4544}p + \frac{\frac{315}{1136}p}{\frac{284}{23}p^2 + 1} \quad k_3 = \frac{2645}{4544}$$

$$Y_4(p) = \frac{\frac{284}{23}p^2 + 1}{\frac{315}{1136}p} = \frac{322624}{7245}p + \frac{1}{\frac{315}{1136}p}$$

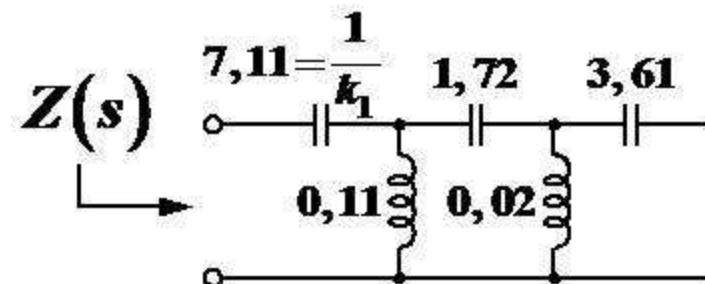
$$k_4 = \frac{322624}{7245}; \quad k_5 = \frac{315}{1136}$$

$$Z(p) = 0,147p + \frac{1}{8,9p + \frac{1}{0,58p + \frac{1}{44,53p + \frac{1}{0,277p}}}}$$

povratak na promenljivu s smenom $s = \frac{1}{p}$:

$$Z(s) = \frac{0,147}{s} + \frac{1}{8,9 + \frac{1}{s + \frac{0,58}{s + \frac{1}{44,53 + \frac{1}{s + \frac{1}{0,277}}}}}}$$

realizacija:



REALIZACIJA ULAZNIH FUNKCIJA RC MREŽE

1. Svi polovi i nule su prosti, negativni i realni.
2. Polovi i nule imaju naizmeničan raspored na negativnom delu realne ose.

Osobine $Z_{RC}(s)$

$$3. Z_{RC}(s) = R_\infty + \frac{1}{C_0 s} + \sum_{i=1}^n \frac{1}{C_i s + \frac{1}{R_i}}$$

Singularitet najbliži $(0,0)$ je pol, a ka ∞ je nula.

$$4. Z_{RC}(s) = \frac{A(s)}{B(s)}; \quad \text{red } A(s) \leq \text{red } B(s) \\ (< \text{ ako je } s = \infty \text{ nula})$$

$$5. Z_{RC}(s) = \frac{(s + \sigma_2)(s + \sigma_4)\dots}{(s + \sigma_1)(s + \sigma_3)\dots} \\ 0 \leq \sigma_1 < \sigma_2 < \sigma_3 < \sigma_4$$

6. Svi ostaci u polovima su pozitivni i realni

7. Ne može imati pol u $s = \infty$ $[Z_{RC}(\infty) = R_\infty]$

8. $Z_{RC}(\sigma)$ je monot. opad. osim u polovima

Mar. 15
OE3LE

Osobine $Y_{RC}(s)$

$$Y_{RC}(s) = C_\infty s + \frac{1}{R_0} + \sum_{i=1}^n \frac{1}{R_i + \frac{1}{C_i s}}$$

Ka ∞ je pol, a singularitet najbliži $(0,0)$ je nula.

$$Y_{RC}(s) = \frac{A(s)}{B(s)}; \quad \text{red } A(s) \geq \text{red } B(s) \\ (> \text{ ako je } s = \infty \text{ pol})$$

$$Y_{RC}(s) = \frac{(s + \sigma_1)(s + \sigma_3)\dots}{(s + \sigma_2)(s + \sigma_4)\dots} \\ 0 \leq \sigma_1 < \sigma_2 < \sigma_3 < \sigma_4$$

Ostatak pola u ∞ je pozitivan i realan, a ostaci u konačnim polovima su neg. i realni

Ne može imati pol u $s = 0$ $[Y_{RC}(0) = 1/R_0]$

$Y_{RC}(\sigma)$ je monot. rastuća osim u polovima

Realizacija se vrši prema sledećem pravilu:

CI:

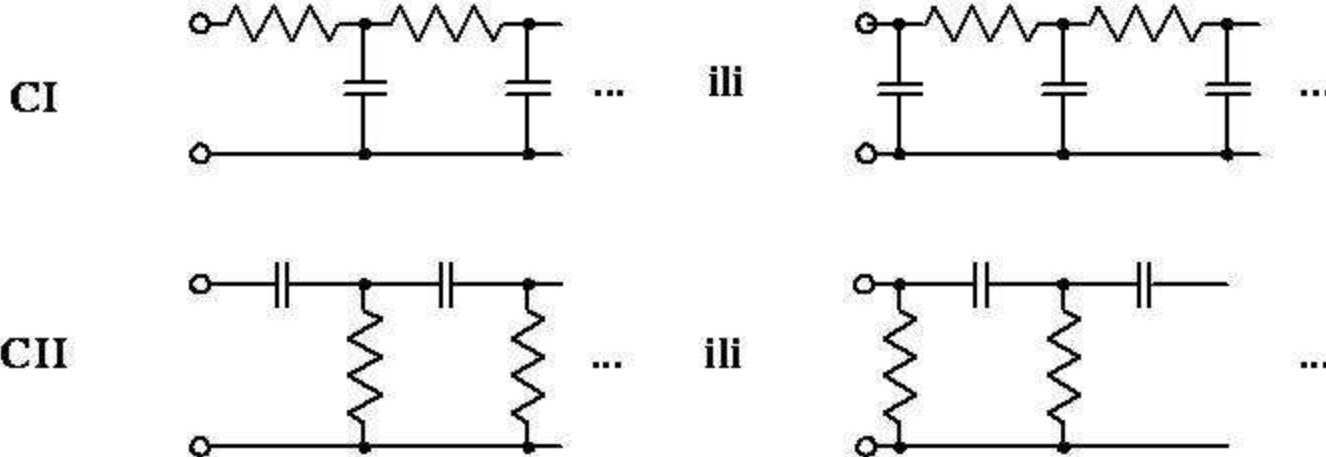
$Y_{RC}(\infty) = \infty \rightarrow$ razvija se $Y_{RC}(s)$ po CI

$Y_{RC}(\infty) \neq \infty \rightarrow$ razvija se $Z_{RC}(s) = 1/Y_{RC}(s)$ po CI

CII:

$Z_{RC}(0) = \infty \rightarrow$ razvija se $Z_{RC}(s)$ po CII

$Z_{RC}(0) \neq \infty \rightarrow$ razvija se $Y_{RC}(s) = 1/Z_{RC}(s)$ po CII



1. Primer: Realizovati u Kauerovoj I formi $Z(s) = \frac{s^2 + 4s + 3}{s^3 + 6s^2 + 8s}$

$$Z(\infty) = 0 \Rightarrow Y(\infty) = \infty \Rightarrow \text{realizuje se } Y(s) = \frac{1}{Z(s)}$$

$$Y_1(s) = \frac{s^3 + 6s^2 + 8s}{s^2 + 4s + 3} \quad C_{\omega 1} = \lim_{s \rightarrow \infty} \left[\frac{1}{s} Y_1(s) \right] = 1$$

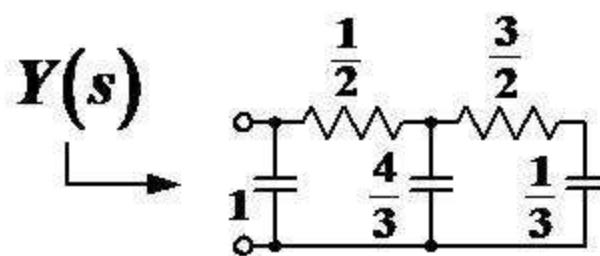
$$Y_1(s) = 1s + \frac{2s^2 + 5s}{s^2 + 4s + 3} = 1s + \frac{1}{Z_1(s)} \Rightarrow Z_1(s) = \frac{s^2 + 4s + 3}{2s^2 + 5s} = \frac{1}{2} + \frac{\frac{3}{2}s + 3}{2s^2 + 5s} \quad R_{\omega 1} = \frac{1}{2}$$

$$Y_2(s) = \frac{2s^2 + 5s}{\frac{3}{2}s + 3} = \frac{4}{3}s + \frac{s}{\frac{3}{2}s + 3} \quad C_{\omega 2} = \frac{4}{3}$$

$$Z_2(s) = \frac{\frac{3}{2}s + 3}{s} = \frac{3}{2} + \frac{3}{s} \quad R_{\omega 2} = \frac{3}{2}; C_{\omega 3} = \frac{1}{3}$$

$$Y(s) = 1s + \frac{1}{\frac{1}{2} + \frac{4}{3}s + \frac{1}{\frac{3}{2}s + \frac{3}{2} + \frac{1}{s}}}$$

Kod realizacije pasivne RC mreže je karakteristično da za svaki pol treba računati po dva koeficijenta (jer R ne daje pol)



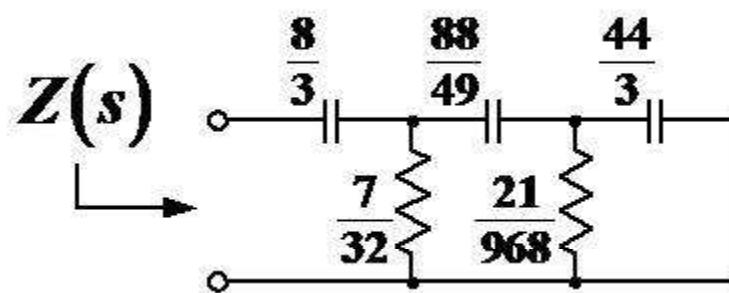
2. Primer: Realizovati u Kauerovoj II formi $Z(s) = \frac{s^2 + 4s + 3}{s^3 + 6s^2 + 8s}$

$Z(0) = \infty \Rightarrow$ realizuje se $Z(s)$

...

Rešenje:

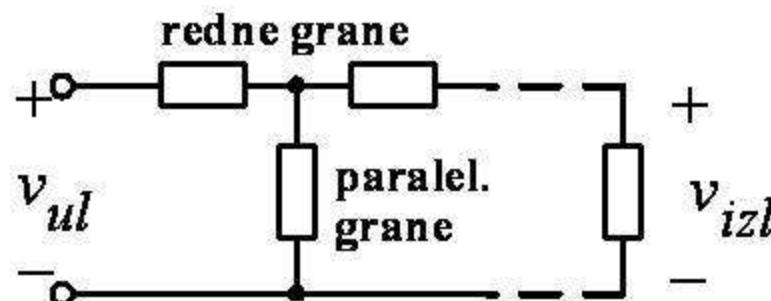
$$Z(s) = \frac{3}{8s} + \frac{1}{\frac{32}{7} + \frac{1}{\frac{49}{38s} + \frac{1}{\frac{968}{21} + \frac{1}{\frac{3}{44s}}}}}$$



PASIVNA REALIZACIJA PRENOSNIH FUNKCIJA

- **lestvičaste strukture** $\left\{ \begin{array}{l} \text{RC - prosti polovi, polovi i nule na neg. delu Re ose} \\ \text{LC - prosti polovi, polovi i nule na imaginarnoj osi} \end{array} \right\}$
- **ukršteni četvoropoli** $\{\text{pogodni za realizaciju } all \text{ pass prenosnih funkcija}\}$
- **Darlingtonova kola** $\{\text{šira klasa prenosnih funkcija, sa kompleksnim polovima}\}$

Lestvičaste mreže



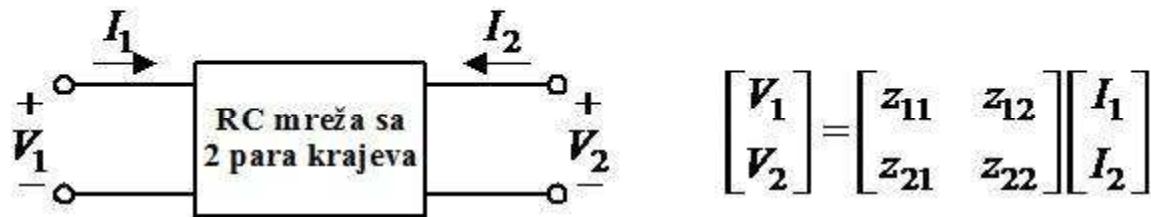
Nula prenosa: vrednost $s = s_k$ za koju je $H(s_k) = 0$, gde je $H(s)$ prenosna funkcija. To je učestanost na kojoj redna grana postaje otvorena veza ili paralelna kratak spoj.

RC lestvičaste mreže

Ako svaka grana RC lestvičaste mreže sadrži samo po jedan element (nule su u 0 ili ∞), prenosna funkcija ima formu:

$$H(s) = \frac{ks^m}{s^n + b_{n-1}s^{n-1} + \dots + b_0} = \frac{ks^m}{B(s)} \quad 0 \leq m \leq n$$

$B(s)$ je polinom n -tog reda sa prostim, realnim, negativnim korenovima.



Za $I_2 = 0$ $H(s) = \left. \frac{V_2}{V_1} \right|_{I_2=0} = \frac{z_{21}}{z_{11}}$

$$\Rightarrow z_{21} = \frac{ks^m}{X(s)} \quad z_{11} = \frac{B(s)}{X(s)} \quad \text{gde je } X(s) \text{ polinom}$$

izabran tako da z_{11} zadovoljava osobine ulazne funkcije RC mreže
(polovi i nule treba da budu prosti i naizmenično poredani na
negativnom delu realne ose, ka $(0,0)$ je pol, a ka ∞ je nula)

$$\text{Slučaj 1: } H(s) = \frac{k}{B(s)}$$

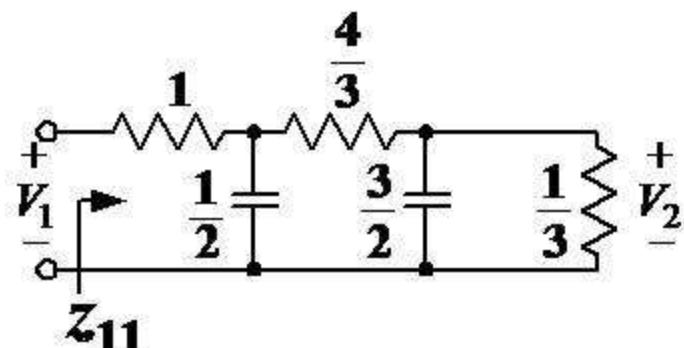
$m = 0$, sve nule prenosa su u $\infty \Rightarrow z_{11}$ treba realizovati u CI formi jer su kod nje kondenzatori u paralelnim granama

$$\text{Primer: } H(s) = \frac{k}{(s+2)(s+4)}$$

$$z_{11} = \frac{(s+2)(s+4)}{(s+1)(s+3)} \quad \begin{array}{l} \leftarrow \text{mora} \\ \leftarrow \text{jedan od mogućih izbora (treba pol da bude najbliži koordinatnom početku i da budu nule i polovi naizmenično poredani)} \end{array}$$

$$z_{11} = \frac{s^2 + 6s + 8}{s^2 + 4s + 3} = 1 + \frac{2s + 5}{s^2 + 4s + 3} = 1 + \frac{1}{\frac{1}{2}s + \frac{2}{3}} = 1 + \frac{1}{\frac{1}{2}s + \frac{2}{3}}$$

$$z_{11} = 1 + \frac{1}{\frac{1}{2}s + \frac{1}{4}} = 1 + \frac{1}{\frac{1}{2}s + \frac{4}{3}} = 1 + \frac{1}{\frac{1}{2}s + \frac{3}{2}} + \frac{1}{\frac{1}{2}s + \frac{1}{3}}$$



Za $V_1(s) = 1$ (Dirakov impuls):

$$V_2(s) = \frac{1}{(s+2)(s+4)} = \hat{H}(s) = \frac{H(s)}{k}$$

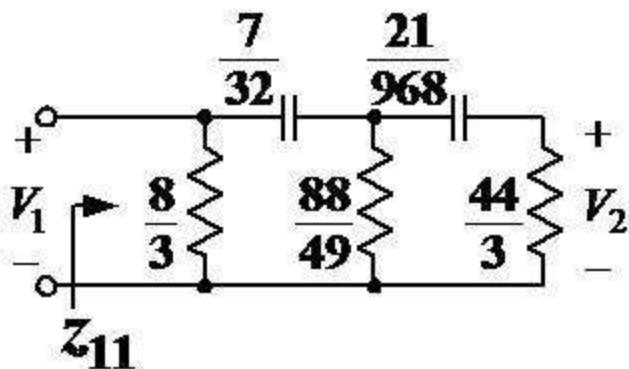
$$\text{Slučaj 2: } H(s) = \frac{ks^n}{s^n + b_{n-1}s^{n-1} + \dots + b_0} = \frac{ks^n}{B(s)}$$

$m = n$, sve nule prenosa su u $(0,0)$ $\Rightarrow z_{11}$ treba realizovati u CII formi jer su kod nje kondenzatori u rednim granama

Primer: $H(s) = \frac{ks^2}{(s+2)(s+4)}$

$$z_{11} = \frac{(s+2)(s+4)}{(s+1)(s+3)} \quad z_{21} = \frac{ks^2}{(s+1)(s+3)}$$

$$z_{11} = \frac{8+6s+s^2}{3+4s+s^2} = \frac{1}{\frac{3}{8} + \frac{32}{7s} + \frac{49}{88} + \frac{1}{21s} + \frac{3}{44}}$$



$$\text{Slučaj 3: } H(s) = \frac{ks^m}{s^n + b_{n-1}s^{n-1} + \dots + b_0} = \frac{ks^m}{B(s)}$$

$m < n$, nule prenosa su i u $(0,0)$ i u $\infty \Rightarrow z_{11}$ treba realizovati delom u CI a preostalim delom u CII formi

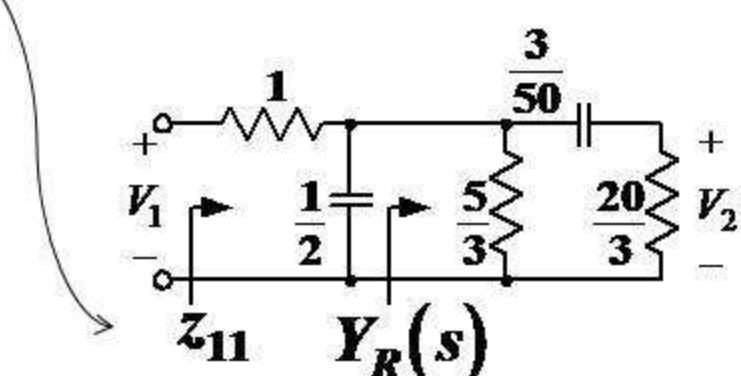
$$\text{Primer: } H(s) = \frac{ks}{(s+2)(s+4)} \quad z_{11} = \frac{(s+2)(s+4)}{(s+1)(s+3)}$$

Jednom primenjujemo CI da izvučemo jednu nulu u $s \rightarrow \infty$, a zatim CII za nulu u $s=0$.

Može da se ide i obrnutim redom: prvo CII pa onda CI

$$z_{11} = \frac{s^2 + 6s + 8}{s^2 + 4s + 3} = 1 + \frac{1}{\frac{3}{2}s + 3} = 1 + \frac{1}{\frac{1}{2}s + Y_R(s)}$$

$$Y_R(s) = \frac{\frac{3}{2}s + 3}{2s + 5} = \frac{3}{5} + \frac{1}{50 + \frac{3}{3s}} = \frac{3}{5} + \frac{1}{20}$$



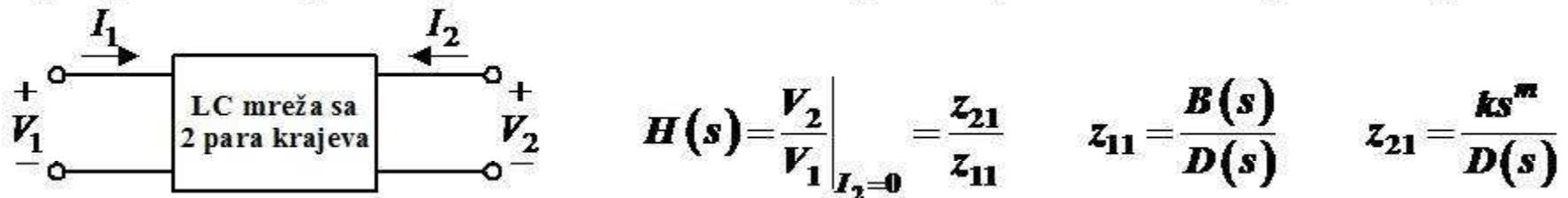
$\left(\text{za CII se ispituje ponašanje } Z_R(0) = \frac{1}{Y_R(0)} \neq \infty \Rightarrow \text{razvija se } Y_R(s) \right)$

LC lestvičaste mreže

Ako svaka grana lestvičaste LC mreže sadrži samo po jedan element, prenosna funkcija ima formu:

$$H(s) = \frac{ks^m}{s^n + b_{n-2}s^{n-2} + \dots + b_0} = \frac{ks^m}{B(s)} \quad 0 \leq m \leq n$$

gde polinom $B(s)$ ima proste korenove na imaginarnoj osi, a m i n su parni brojevi.



gde je $D(s)$ polinom $n-1$ reda sa prostim čisto imaginarnim korenovima koji su u alternaciji sa korenovima $B(s)$

Slučaj 1: $m=0$ $H(s) = \frac{k}{B(s)} \Rightarrow \text{CI}$

Slučaj 2: $m=n$ $H(s) = \frac{ks^n}{B(s)} \Rightarrow \text{CII}$

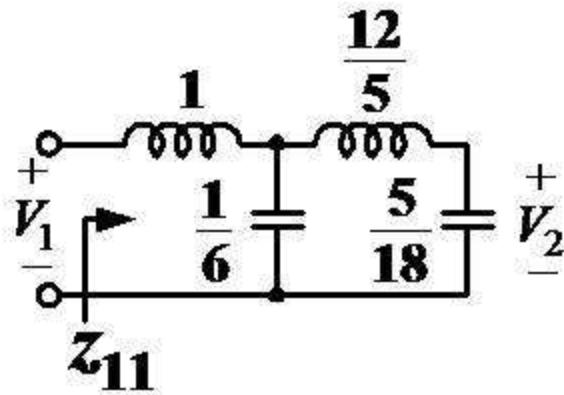
Slučaj 3: $0 < m < n$ $H(s) = \frac{ks^m}{B(s)} \Rightarrow \begin{cases} m \text{ puta CII} \\ (n-m) \text{ puta CI} \end{cases}$

Primer: $H(s) = \frac{k}{(s^2 + 1)(s^2 + 9)}$

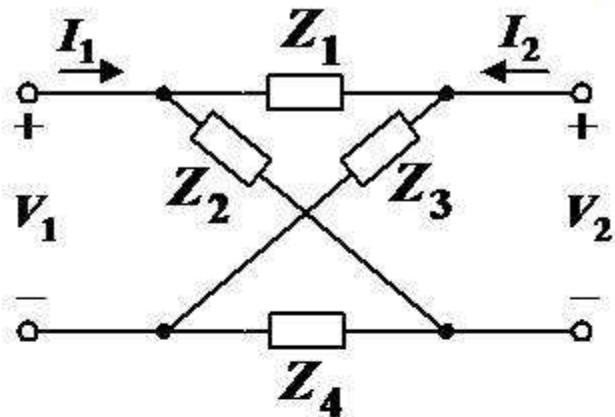
$$z_{11}(s) = \frac{(s^2 + 1)(s^2 + 9)}{s(s^2 + 4)} \quad z_{21}(s) = \frac{k}{s(s^2 + 4)}$$

$$z_{11}(s) = s + \frac{1}{\frac{1}{6}s + \frac{12}{5}} \quad \Rightarrow \quad H(s) = \frac{9}{(s^2 + 1)(s^2 + 9)}$$

$$\frac{1}{6}s + \frac{12}{5} = \frac{5}{18}s$$



Ukršteni četvoropol



Ako nema ograničenja u izboru elemenata, onda se ovom strukturom može realizovati svaka prenosna funkcija.

Ako uzmemо simetričnu ukrštenу čeliju:

$$Z_a = Z_1 = Z_4 \quad Z_b = Z_2 = Z_3$$

$$\left. \begin{array}{l} V_1 = \frac{Z_a + Z_b}{2} I_1 + \frac{Z_b - Z_a}{2} I_2 \\ V_2 = \frac{Z_b - Z_a}{2} I_1 + \frac{Z_a + Z_b}{2} I_2 \end{array} \right\} \quad H(s) = \frac{V_2}{V_1} \Big|_{I_2=0} = \frac{Z_b - Z_a}{Z_b + Z_a}$$

All-pass filter ima $H(s) = \frac{p(-s)}{p(s)}$ gde je $p(s)$ Hurvicov polinom.

$$p(s) = m(s) + n(s) = \text{paran} + \text{neparan deo } p(s)$$

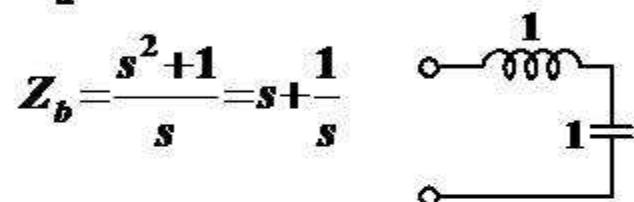
$$H(s) = \frac{m(s) - n(s)}{m(s) + n(s)} = \frac{\frac{m(s)}{n(s)} - 1}{\frac{m(s)}{n(s)} + 1} = \frac{1 - \frac{n(s)}{m(s)}}{1 + \frac{n(s)}{m(s)}}$$

$H(s)$ se može realizovati tako da je Z_a (ili Z_b) = 1Ω , a Z_b (Z_a) je LC mreža (ulazna funkcija LC mreže)

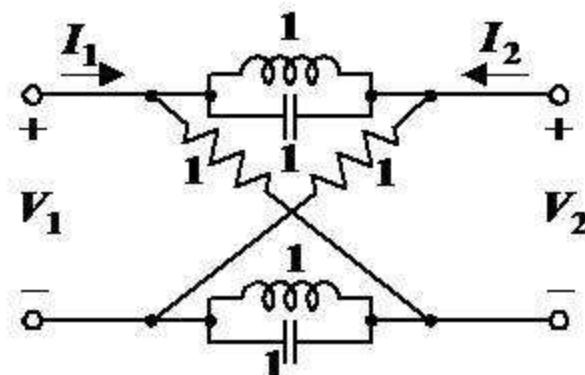
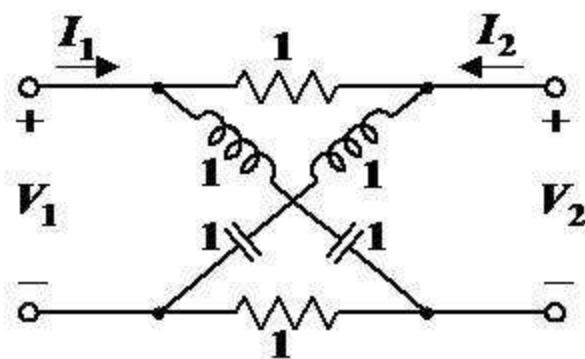
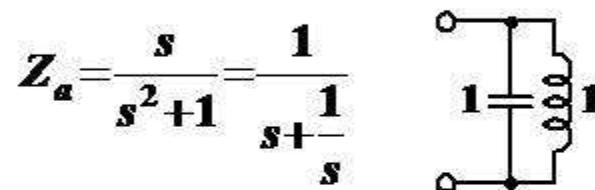
Primer:

$$H(s) = \frac{s^2 - s + 1}{s^2 + s + 1} = \frac{\frac{s^2 + 1}{s} - 1}{\frac{s^2 + 1}{s} + 1} = \frac{1 - \frac{s}{s^2 + 1}}{1 + \frac{s}{s^2 + 1}}$$

1º $Z_a = 1\Omega$

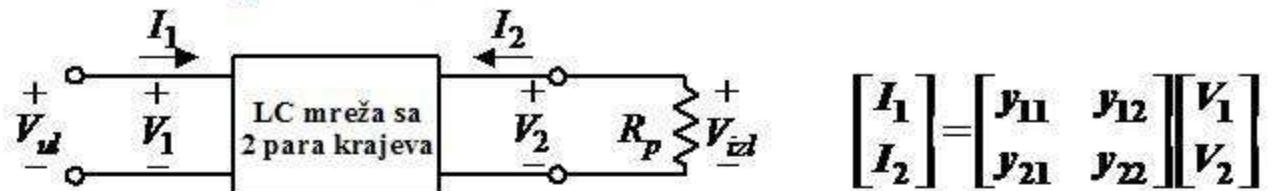


2º $Z_b = 1\Omega$



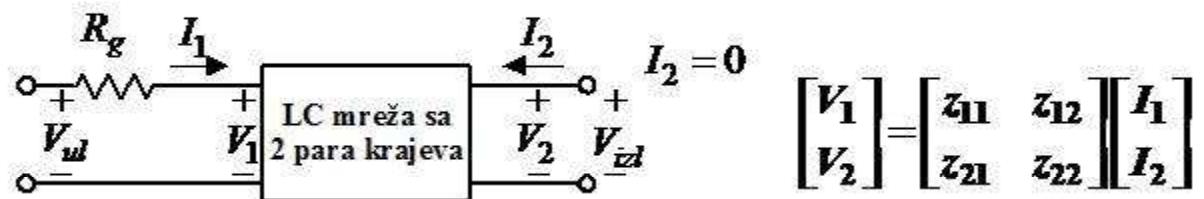
Realizacija prenosne funkcije pomoću četvoropola bez gubitaka zatvorenog otpornicima

Darlingtonov metod



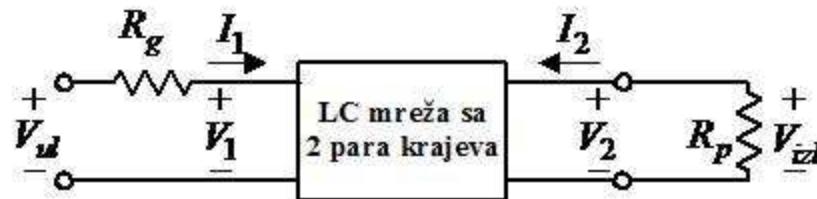
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$H(s) = \frac{V_{z_l}}{V_{ml}} = \frac{-y_{21}}{\frac{1}{R_p} + y_{22}}$$



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$H(s) = \frac{V_{z_l}}{V_{ml}} = \frac{z_{12}}{R_g + z_{11}} \quad (z_{21} = z_{12})$$



$$H(s) = \frac{V_{z_l}}{V_{ml}} = \frac{-y_{12}}{\frac{1}{R_p} + y_{22}} \cdot \frac{1}{R_g \left[y_{11} - \frac{y_{12}^2}{\frac{1}{R_p} + y_{22}} \right] + 1}$$

Četvoropol bez gubitaka sa jednim otpornikom opterećenja

y_{12} , z_{12} , kao i y_{22} , z_{11} (ulazne funkcije LC mreže), predstavljaju količnike parnog i neparnog polinoma

$$H(s) = \frac{A(s)}{B(s)} = \frac{M_1(s)}{M_2(s) + N_2(s)} = \frac{\frac{M_1(s)}{N_2(s)}}{\frac{M_2(s)}{N_2(s)} + 1} \quad \text{ili}$$

$$H(s) = \frac{A(s)}{B(s)} = \frac{N_1(s)}{M_2(s) + N_2(s)} = \frac{\frac{N_1(s)}{M_2(s)}}{1 + \frac{N_2(s)}{M_2(s)}} \quad \begin{cases} N(s) \text{ je neparan, a } M(s) \\ \text{je paran deo polinoma} \end{cases}$$

Problem se svodi na simultanu realizaciju y_{22} i y_{12} , odnosno z_{11} i z_{12} , pri čemu je $R_g = 1\Omega$ ili $R_p = 1\Omega$. (Za $A(s) = ks^n$ problem se svodi na realizaciju lestvičaste LC mreže bez opterećenja.)

Primer 1

$$H(s) = \frac{V_{izl}}{V_{ul}} = \frac{1}{s^2 + s + 1}$$

Realizovati zadatu prenosnu funkciju sa četvoropolom bez gubitaka zatvorenim otpornikom od 1Ω .

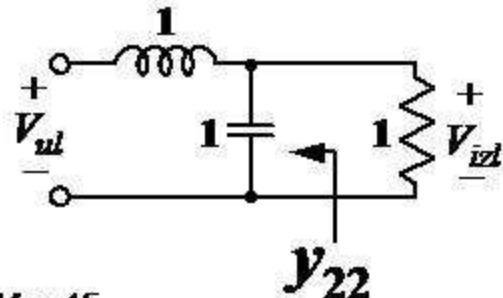
$$H(s) = \frac{\frac{1}{s}}{\frac{s^2 + 1}{s} + 1} \Rightarrow \begin{cases} y_{21} = -\frac{1}{s} \\ y_{22} = \frac{s^2 + 1}{s} = s + \frac{1}{s} \end{cases}$$



jer je brojilac paran polinom.

Nule prenosa su u ∞ , pa y_{22} razvijamo po CI.

y_{22} se realizuje sa desna na levo i pri tome poslednji element ostaje sa otvorenim krajem (pri određivanju y_{22} se V_{ul} kratko spaja)



$$H(s) = \frac{V_{izl}}{V_{ul}} = \frac{\frac{1}{s}}{\frac{1 + \frac{1}{s}}{s + \frac{1}{s + \frac{1}{s}}}} = \frac{\frac{1}{s}}{\frac{1}{s + 1} + \frac{1}{s}} = \frac{1}{s^2 + s + 1}$$

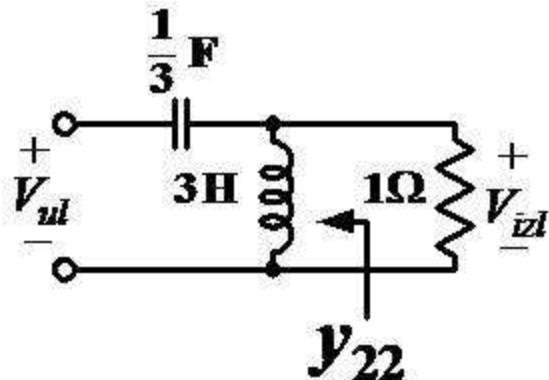
Primer 2

Realizovati $H(s) = \frac{V_{izl}}{V_{ul}} = \frac{s^2}{s^2 + 3s + 1}$ preko LC četvoropola zatvorenog otpornikom $R_p = 1\Omega$

$$H(s) = \frac{\frac{s^2}{3s}}{\frac{s^2 + 1}{3s} + 1} \Rightarrow \begin{cases} y_{12} = \frac{s}{3} \\ y_{22} = \frac{s^2 + 1}{3s} \end{cases}$$

Nule prenosa su u 0, pa y_{22} razvijamo po CIL.

$$y_{22} = \frac{1}{3s} + \frac{1}{s}$$



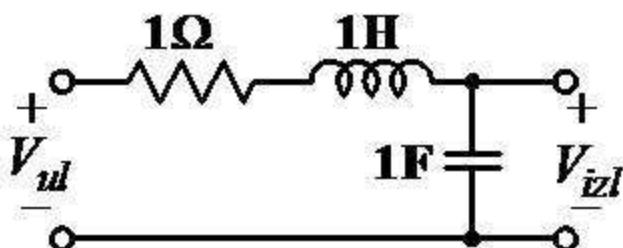
Primer 3

Realizovati $H(s) = \frac{V_{izl}}{V_{ul}} = \frac{1}{s^2 + s + 1}$ preko LC četvoropola zatvorenog otpornikom $R_g = 1\Omega$ na ulazu.

$$H(s) = \frac{\frac{1}{s}}{\frac{s^2 + 1}{s} + 1} \Rightarrow \begin{cases} z_{12} = \frac{1}{s} \\ z_{11} = \frac{s^2 + 1}{s} \end{cases}$$

Nule prenosa su u ∞ , pa z_{11} razvijamo po CL.

$$z_{11} = s + \frac{1}{s}$$



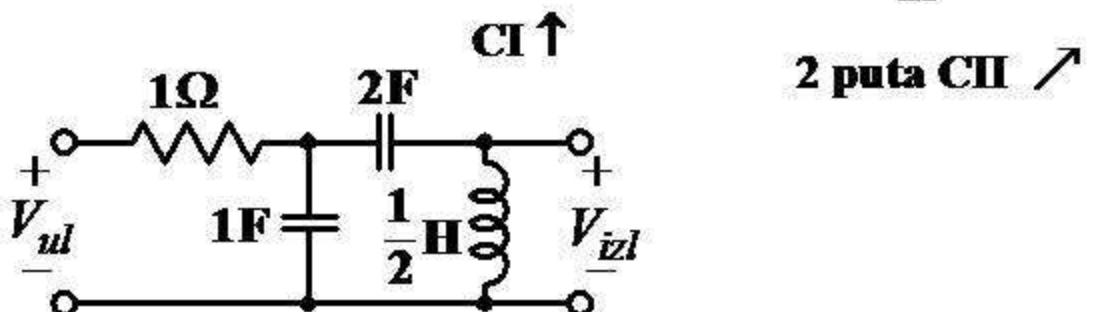
Primer 4

Realizovati $H(s) = \frac{V_{izl}}{V_{ul}} = \frac{s^2}{s^3 + s^2 + 3s + 1}$ preko LC četvoropola zatvorenog otpornikom $R_g = 1\Omega$ na ulazu.

$$H(s) = \frac{\frac{s^2}{s^3 + 3s}}{\frac{s^2 + 1}{s^3 + 3s} + 1} \Rightarrow \begin{cases} z_{12} = \frac{s^2}{s^3 + 3s} \\ z_{11} = \frac{s^2 + 1}{s^3 + 3s} \end{cases}$$

2 nule prenosa su u $s = 0$, pa dva puta izdvajamo pol z_{11} u koordinatnom početku (razvijamo po CII), a treća nula prenosa je u ∞ , pa jedan pol izdvajamo po CI.

$$z_{11} = \frac{s^2 + 1}{s^3 + 3s} = \frac{1}{s^3 + 3s} = \frac{1}{s + \frac{2s}{s^2 + 1}} = \frac{1}{s + \frac{1}{\frac{s^2 + 1}{2s}}} = \frac{1}{s + \frac{1}{\frac{1}{2s} + \frac{1}{2}}} = \frac{1}{s}$$



Primer 5

Realizovati $H(s) = \frac{V_{izl}}{V_{ul}} = \frac{s^3}{s^3 + s^2 + 3s + 1}$ preko LC četvoropola zatvorenog otpornikom $R_g = 1\Omega$ na ulazu.

$$H(s) = \frac{\frac{s^3}{s^2 + 1}}{\frac{s^3 + 3s}{s^2 + 1} + 1} \Rightarrow \begin{cases} z_{12} = \frac{s^3}{s^2 + 1} \\ z_{11} = \frac{s^3 + 3s}{s^2 + 1} \end{cases}$$

sve 3 nule prenosa su u $s = 0$, pa 3 puta izdvajamo pol z_{11} u koordinatnom početku (razvijamo po CII).

$$\frac{1}{z_{11}} = \frac{s^2 + 1}{s^3 + 3s} = \frac{1}{3s} + \frac{\frac{2}{3}s^2}{s^3 + 3s} = \frac{1}{3s} + \frac{1}{2s} + \frac{1}{3s}$$

