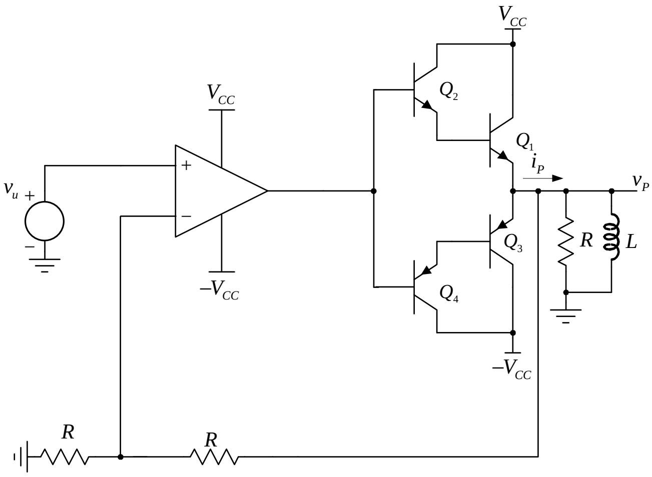
Zadatak. U kolu sa slike operacioni pojačavač se može smatrati idealnim, osim što ima strujno ograničenje $i_{OP\max} = 5\text{mA}$. Parametri tranzistora su $\beta_{FN} = 100$, $\beta_{FP} = 30$, $|V_{BE}| = 0.7\text{V}$, dok je $V_{CC} = 15\text{V}$, $R = 20\Omega$, $L = 1\text{mH}$. Napon na ulazu kola je trougaonog talasnog oblika amplitude V_u i učestanosti f.



a) Ako je $V_u = 5\text{V}$ i $f = 10\text{kHz}$, nacrtati i označiti dijagrame v_U, v_P, i_R, i_L, i_P, i_{C1}, i_{C2} i v_{IOP} tokom jedne periode izlaznog napona.

b) Ako je $V_u = 5\text{V}$ i $f = 10\text{kHz}$, odrediti srednju snagu disipacije tranzistora Q_1

c) odrediti zavisnost maksimalne amplitude ulaznog napona V_u od T

d) ukoliko $L \to \infty$, dimenzionisati tranzistore Q_1 i Q_2 po snazi.

Rešenje:

a)

$$v_U = 5\text{V}\,\Delta(t)$$

$$v_P = 2v_U = 10\,\text{V}\Delta(t)$$

$$i_R = \frac{v_P}{R} = \frac{2v_U}{R} = 0.5\,\text{A}\Delta(t)$$

$$i_L = \frac{1}{L}\int v_P dt = \frac{2V_u}{L}\int \Delta(t)\,dt = \frac{2V_u}{L}\frac{T}{8}\left(1-\Delta^2(t)\right)\Pi(t) = \frac{V_u T}{4L}\left(1-\Delta^2(t)\right)\Pi(t) =$$
$$= 0.125\,\text{A}\left(1-\Delta^2(t)\right)\Pi(t)$$

$$i_P = i_R + i_L = \frac{2V_u}{R}\Delta(t) + \frac{V_u T}{4L}\left(1-\Delta^2(t)\right)\Pi(t) =$$
$$= 0.5\,\text{A}\Delta(t) + 0.125\,\text{A}\left(1-\Delta^2(t)\right)\Pi(t)$$

$$T = \frac{1}{f} = 100\mu\text{s}$$

Tranzistor Q_1 vodi dok je $i_P > 0$, tj. gasi se u trenutku t_1, kada je $\Delta(t) = \Delta(t_1)$:

$$i_P\left(\Delta(t_1)\right) = \frac{2V_u}{R}\Delta(t_1) + \frac{V_u T}{4L}\left(1-\Delta^2(t_1)\right)\Pi(t_1) = 0,$$

Kako se radi u prvoj poluperiodi, to je $\Pi(t_1) = 1$, odnosno

$$\frac{T}{4L}\Delta^2(t_1) - \frac{2}{R}\Delta(t_1) - \frac{T}{4L} = 0$$

$$t_1 = \frac{T}{4} - \left(\pm\sqrt{\left(\frac{L}{R}\right)^2 + \left(\frac{T}{4}\right)^2} - \frac{L}{R}\right)$$

Kako mora biti $t_1 > \frac{T}{4}$, bira se znak “–“:

$$t_1 = \frac{T}{4} + \sqrt{\left(\frac{L}{R}\right)^2 + \left(\frac{T}{4}\right)^2} - \frac{L}{R}$$

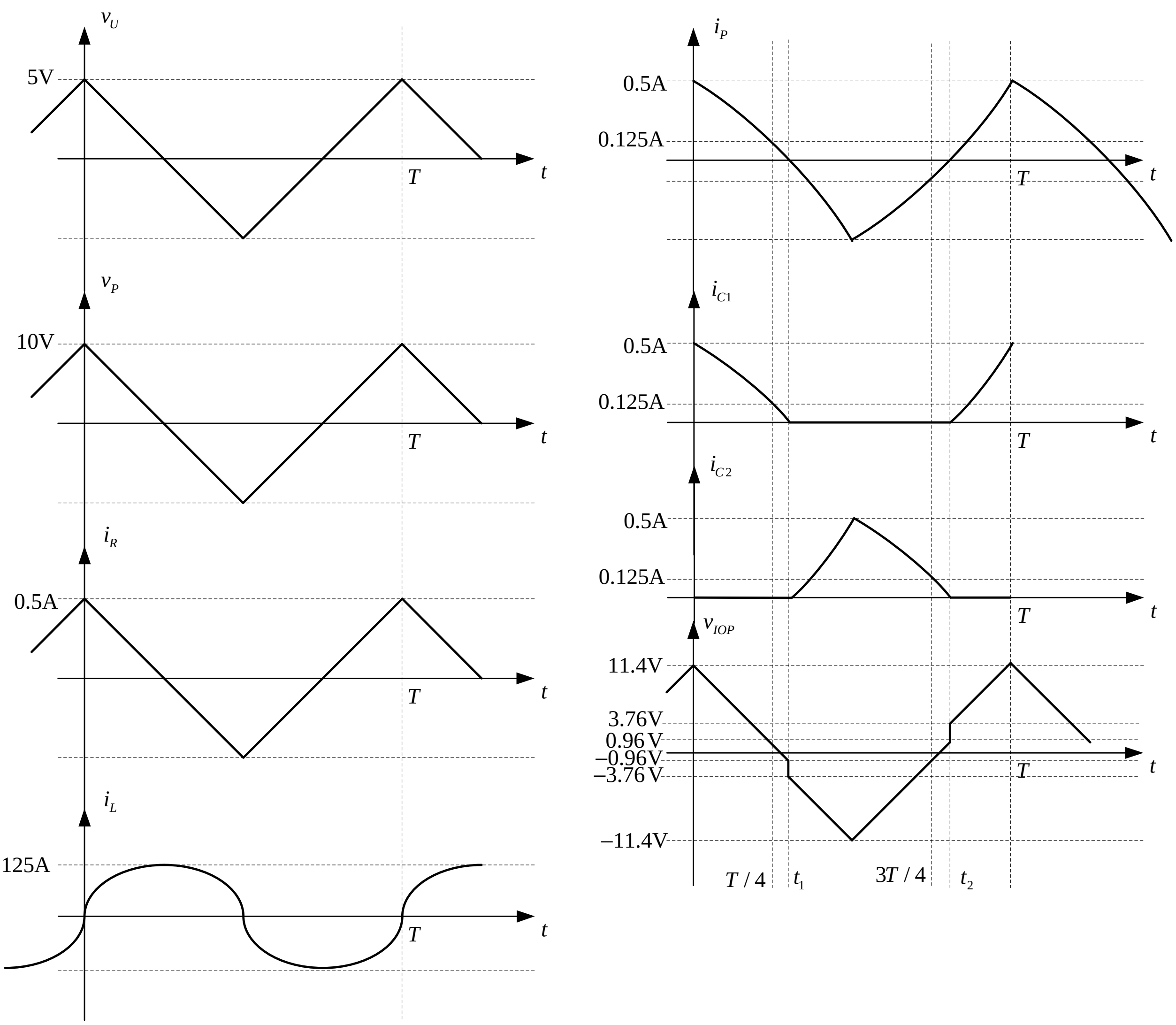
$$t_1 = 31\,\mu\text{s}$$

$$v_P\left(t_1\right) = 10\,\text{V}\Delta\left(t_1\right) = -2.36\,\text{V}$$

Tranzistor Q_1 se ponovo uključuje kada ponovo postane $i_P > 0$, odnosno (zbog simetrije)

$$t_2 = \frac{T}{2} + t_1 = 81\,\mu\text{s}$$

$$v_{IOP} = \begin{cases} v_P + 2V_{BE} = 10\text{ V}\Delta(t) + 1.4\text{ V} & i_P > 0 \\ v_P - 2V_{BE} = 10\text{ V}\Delta(t) - 1.4\text{ V} & i_P < 0 \end{cases}$$



b)

$$p_{D1} = v_{CE1} i_{C1} = \begin{cases} (V_{CC} - v_P) i_P, & i_P > 0 \\ 0, & i_P \le 0 \end{cases}$$

$$P_{D1} = \frac{1}{T}\int_{nT}^{nT+t_1} (V_{CC} - v_P) i_P dt + \frac{1}{T}\int_{t_2}^{(n+1)T} (V_{CC} - v_P) i_P dt =$$

$$= \frac{1}{T}\int_{nT}^{nT+t_1} \left(15 - 10\,\mathrm{V}\Delta(t)\right)\left(0.5\,\mathrm{A}\Delta(t) + 0.125\,\mathrm{A}\left(1 - \Delta^2(t)\right)\Pi(t)\right) dt +$$

$$+ \frac{1}{T}\int_{nT+t_2}^{(n+1)T} \left(15 - 10\,\mathrm{V}\Delta(t)\right)\left(0.5\,\mathrm{A}\Delta(t) + 0.125\,\mathrm{A}\left(1 - \Delta^2(t)\right)\Pi(t)\right) dt$$

$$P_{D1} = \frac{1}{T}\int_{nT}^{nT+t_1} \left(15 - 10\,\mathrm{V}\Delta(t)\right)\left(0.5\,\mathrm{A}\Delta(t) + 0.125\,\mathrm{A}\left(1 - \Delta^2(t)\right)\right) dt +$$

$$+ \frac{1}{T}\int_{nT+t_2}^{(n+1)T} \left(15 - 10\,\mathrm{V}\Delta(t)\right)\left(0.5\,\mathrm{A}\Delta(t) - 0.125\,\mathrm{A}\left(1 - \Delta^2(t)\right)\right) dt$$

$$P_{D1} = \frac{1}{T}\int_{0}^{t_1} \left(15 - 10\left(1 - 4\frac{t}{T}\right)\right)\left(0.5\,\mathrm{A}\left(1 - 4\frac{t}{T}\right) + 0.125\,\mathrm{A}\left(1 - \left(1 - 4\frac{t}{T}\right)^2\right)\right) dt +$$

$$+ \frac{1}{T}\int_{t_2}^{T} \left(15 - 10\,\mathrm{V}\left(-3 + 4\frac{t}{T}\right)\right)\left(0.5\,\mathrm{A}\left(-3 + 4\frac{t}{T}\right) - 0.125\,\mathrm{A}\left(1 - \left(-3 + 4\frac{t}{T}\right)^2\right)\right) dt$$

Smena $u = \dfrac{t}{T}$:

$$P_{D1} = \int_{0}^{t_1/T} \left(15 - 10(1 - 4u)\right)\left(0.5\,\mathrm{A}(1 - 4u) + 0.125\,\mathrm{A}\left(1 - (1 - 4u)^2\right)\right) du +$$

$$+ \int_{t_2/T}^{1} \left(15 - 10(-3 + 4u)\right)\left(0.5\,\mathrm{A}(-3 + 4u) - 0.125\,\mathrm{A}\left(1 - (-3 + 4u)^2\right)\right) du =$$

$$= \int_{0}^{t_1/T} (5 + 40u)\left(0.5 - u - 2u^2\right) du + \int_{t_2/T}^{1} (45 - 40u)\left(-0.5 - u + 2u^2\right) du = \ldots = 1.15\,\mathrm{W}$$

c)

naponsko ograničenje izlaza operacionog pojačavača: $V_{u\max} = \dfrac{V_{CC} - 2V_{BE}}{2} = 6.8\mathrm{V}$

strujno ograničenje se javlja kada je $i_{P\max} = (1 + \beta_{FP})^2 i_{OP\max} = 4.805\mathrm{A}$ (gledamo i_{C2}, zbog manjeg strujnog pojačanja pnp tranzistora)

Da li je maksimalna jačina struje kroz potrošač $\dfrac{2V_u}{R}$, kako se čini sa ***skiciranog*** dijagrama?

$$i_P = \frac{2V_u}{R}\Delta(t) + \frac{V_u T}{4L}\left(1 - \Delta^2(t)\right)\Pi(t)$$

Posmatramo prvu poluperiodu, kada je $\Pi(t) = 1$, $\Delta(t) = 1 - 4\dfrac{t}{T}$

$$i_P = \frac{2V_u}{R}\Delta(t) + \frac{V_u T}{4L}\left(1 - \Delta^2(t)\right)$$

Pitanje je da li ova funkcija ima maksimum na intervalu $(0,t_1)$?

$$\frac{di_P}{dt}=\frac{2V_u}{R}\Delta'(t)-\frac{2V_uT}{4L}\Delta(t)\Delta'(t)=0$$

$$\frac{d^2i_P}{dt^2}=-\frac{2V_uT}{4L}(\Delta'(t))^2<0$$

Kako je drugi izvod uvek negativan, funkcija može imati samo maksimum.

$$\frac{1}{R}-\frac{T}{4L}\Delta(t)=0$$

$$\Delta(t)=\frac{4L}{RT},\ 1-4\frac{t}{T}=\frac{4L}{RT}$$

$$t=\frac{T}{4}-\frac{L}{R}$$

Da bi postojao maksimum na intervalu, mora biti $t_1>t>0$, odakle se dobija

$$T>\frac{4L}{R}=200\mu\text{s}$$

Minimum struje potrošača (maksimum struje tranzistora Q_2) je odavde

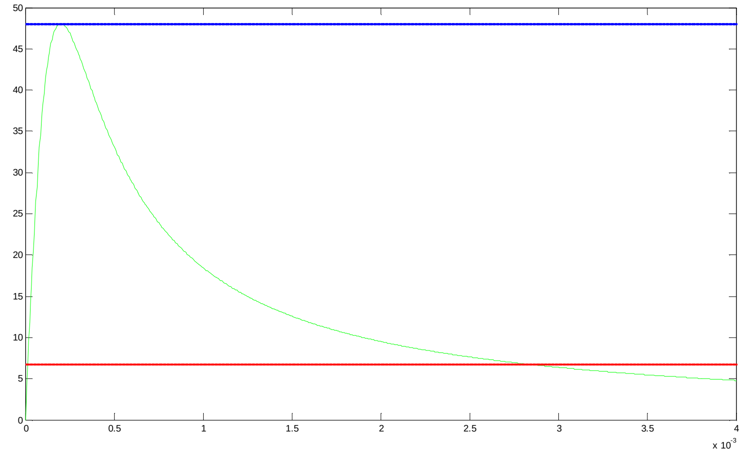
$$i_{P\max}=\begin{cases}\frac{V_u}{L}\left(\left(\frac{L}{R}\right)^2\frac{4}{T}+\frac{T}{4}\right) & T>200\ \mu s\\ \frac{2V_u}{R} & T<200\ \mu s\end{cases}$$

$$V_u=\begin{cases}\frac{i_{P\max}L}{\left(\frac{L}{R}\right)^2\frac{4}{T}+\frac{T}{4}} & T>200\ \mu s\\ \frac{Ri_{P\max}}{2}=48.05\,V & T<200\ \mu s\end{cases}$$

Uzima se strožiji od dva uslova. Za male vrednosti T to je naponsko, za velike strujno, granična vrednost periode se određuje na osnovu jednačine

$$\frac{i_{P\max}L}{\left(\frac{L}{R}\right)^2\frac{4}{T}+\frac{T}{4}}=6.8\,V,\ T=2.8\text{ms}.$$

$$V_u=\begin{cases}6.8\,V & T<2.8\ ms\\ \frac{i_{P\max}L}{\left(\frac{L}{R}\right)^2\frac{4}{T}+\frac{T}{4}} & T>2.8\ ms\end{cases}$$



d)

$$P_{D1} = \frac{1}{T}\left(\int_0^{T/4} \left(V_{CC} - v_P\right) i_R dt + \int_{3T/4}^{T} \left(V_{CC} - v_P\right) i_R dt\right)$$

$$P_{D1} = \frac{1}{T}\left(\int_0^{T/4} \left(V_{CC} - 2V_u\Delta(t)\right)\left(\frac{2V_u}{R}\Delta(t)\right)dt + \int_{3T/4}^{T} \left(V_{CC} - 2V_u\Delta(t)\right)\left(\frac{2V_u}{R}\Delta(t)\right)dt\right) =$$

$$= \frac{1}{T}\left(V_{CC}\frac{2V_u}{R}\left(\int_0^{T/4} \Delta(t)\, dt + \int_{3T/4}^{T} \Delta(t)\, dt\right) - \frac{4V_u^{\ 2}}{R}\left(\int_0^{T/4} \Delta^2(t)\, dt + \int_{3T/4}^{T} \Delta^2(t)\, dt\right)\right)$$

$$P_{D1} = \frac{1}{T}\left(7.5\,\mathrm{W}\left(\int_0^{T/4} \Delta(t)\, dt + \int_{3T/4}^{T} \Delta(t)\, dt\right) - 5\,\mathrm{W}\left(\int_0^{T/4} \Delta^2(t)\, dt + \int_{3T/4}^{T} \Delta^2(t)\, dt\right)\right) = \ldots$$

Može direktno, ali može i... Zakon o održanju energije:

$$P_{D1} = \frac{2P_{CC} - P_{OUT}}{2}$$

$$P_{CC} = V_{CC} I_{C1} = V_{CC}\overline{i_{C1}} = \ldots = \text{< sa slike >} = V_{CC}\frac{2V_u}{4R} = \frac{V_{CC}}{2R}V_u$$

$$P_{OUT} = \frac{\overline{v_P^{\ 2}}}{R} = \frac{\left(2V_u\right)^2 \overline{\Delta^2(t)}}{R} = \frac{4V_u^{\ 2}}{3R}$$

$$P_{D1} = \frac{2\frac{V_{CC}}{2R}V_u - \frac{4}{3R}V_u^{\ 2}}{2} = \frac{V_{CC}}{2R}V_u - \frac{2}{3R}V_u^{\ 2} = \frac{1}{R}V_u\left(\frac{V_{CC}}{2} - \frac{2V_u}{3}\right)$$

Maksimum ove funkcije...

$$\frac{dP_{D1}}{dV_u} = \frac{V_{CC}}{2R} - \frac{4V_u}{3R} = 0$$

$$V_u = \frac{3V_{CC}}{8} = 5.625\mathrm{V}$$

$$P_{D1\max} = 1.05\mathrm{W}$$