10.[1] Применом својстава парних и непарних сигнала израчунати вредност интеграла

(а) *(јул 2021, К-П1а)* $I_0 = \int_{-\pi/2}^{\pi/2} \frac{\cos t}{1+e^{\sin 2t}} dt$;

(б) *(јун 2021, К-П3б)* $I_0 = \int_{-\pi/3}^{\pi/3} \frac{1 - t + 2t^3 - t^5 + 2t^7}{\cos^2(t)} dt$.

$I_\phi = \int_{-\pi/3}^{\pi/3} \frac{dt}{\cos^2 t}$ Ev $\frac{1}{\cos^2 t}$

$= 2 \int_\phi^{\pi/3} \frac{dt}{\cos^2 t} = 2 \tan t \Big|_\phi^{\pi/3} = 2\tan\frac{\pi}{3}$

Периодичност сигнала

11.[2] Утврдити да ли су следећи сигнали периодични и за оне који јесу израчунати основни период:

(а) $x(t) = \cos(3t) + \sin(5t)$;   (б) $x(t) = \cos(6t) + \sin(\pi t)$;   (в) $x(t) = \cos(6t) + \sin(8t) + e^{j2t}$.

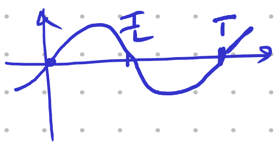
$\omega_1 = 6$ $\omega_2 = \pi$

12. Реална дискретна синусоида дефинисана је у облику $x[n] = A\cos(\Omega_0 n + \phi)$, где је $A \geqslant 0$, $|\Omega_0| \leqslant \pi$ и $|\phi| \leqslant \pi$. Ако дата секвенца

(а) $\{0,1,0,-1\}$;   (б) $\{0,1,1,0,-1,-1\}$;   (в) $\{1,0,-1,-\sqrt{2},-1,0,1,\sqrt{2}\}$

представља основни период ове синусоиде, при чему је први члан $x[0]$, одредити параметре A, Ω_0 и ϕ.

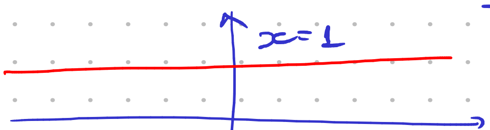
11° $x(t) = x(t+T)$, $\forall t$, периодичан са периодом T

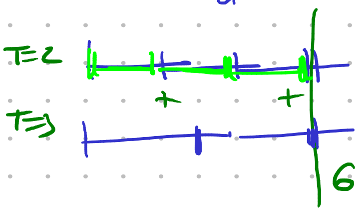


$x(t) = \sin(\omega t)$ основни период $T_\phi = \min T > 0$

$T = 2\pi/\omega$.

$x(t) = \cos(\omega t)$, $\omega \to \phi$, $T \to \infty$





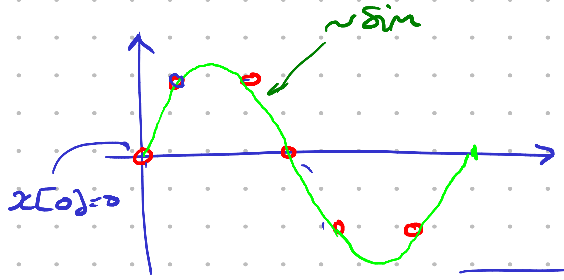
$x(t) = x_1(t) + x_2(t)$

НЗС(T_1, T_2) T_1 T_2

(a) $\cos(3t) + \sin(5t)$, $T_1 = \frac{2\pi}{3}$;   НЗС$\left(\frac{2\pi}{3}, \frac{2\pi}{5}\right) = 2\pi$ НЗС$\left(\frac{1}{3}, \frac{1}{5}\right) = 2\pi$

$\omega_1 = 3$ $\omega_2 = 5$ $T_2 = \frac{2\pi}{5}$, $T = 2\pi$ $\boxed{\omega = 1}$

12° $x[n] = A_\phi \cos(\Omega_\phi n + \phi)$



$x[\phi] = A_\phi \cos(\phi) = \phi \Rightarrow \boxed{x[n] = A_\phi \sin(\Omega_\phi n)}$

$\cos\phi = \phi \Rightarrow \phi = \pm\frac{\pi}{2}$.

$\boxed{\phi = -\frac{\pi}{2}}$

$x[n] = A_\phi \sin\left(\frac{n\pi}{3}\right)$

$n=1 \Rightarrow x[1] = 1 = A_\phi \sin\frac{\pi}{3}$

$\boxed{\Omega_\phi = \frac{2\pi}{N} = \frac{2\pi}{6} = \frac{\pi}{3}}$

$\boxed{A_\phi = \frac{2}{\sqrt{3}}}$.

14. Извести израз за снагу сигнала:

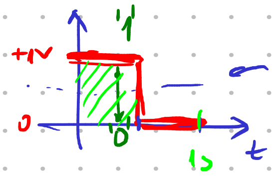
$$T = \frac{2\pi}{\omega_0}$$

$$x(t) = a_0 + \boxed{a_1 \cos(\omega_0 t)} + a_2 \cos(2\omega_0 t) + \cdots + a_n \cos(n\omega_0 t), \qquad (n \in \mathbb{N})$$

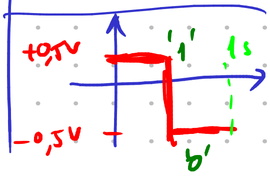
где су $a_1, a_2, \ldots, a_n$ познате реалне константе.

$x(t)$, Енергија $W = \int_{-\infty}^{\infty} x^2(t)\,dt$,

Ср. снага $P = \lim_{T\to\infty} \frac{1}{T}\int_{-T}^{T} x^2(t)\,dt = \frac{1}{T_\phi}\int_{\langle T_\phi \rangle} x^2(t)\,dt$



$\leftarrow (1V)^2 \cdot \frac{1}{2}s = \frac{1}{2}V^2 s$



$\leftarrow \left(\frac{1}{2}V\right)^2 \cdot 1s = \frac{1}{4}V^2 s$

(14) $x(t) = a_0 + a_1 \cos(\boxed{\omega_0 t}) + a_2 \cos(2\boxed{\omega_0 t}) + \cdots$

$x(\phi) = a_0 + a_1 \cos(\boxed{\phi}) + a_2 \cos(2\boxed{\phi}) + \cdots + a_n \cos(n\boxed{\phi})$

$\phi = \omega_0 t$, $T = 2\pi$

$P = \frac{1}{2\pi}\int_0^{2\pi} x^2(\phi)\,d\phi = a_0^2 + a_1^2 \cos^2(\phi) + a_2^2 \cos^2(2\phi) + \cdots + a_n^2 \cos^2(n\phi)$

$+ a_0 a_1 \cos(\phi) + a_0 a_2 \cos(2\phi) + \cdots +$

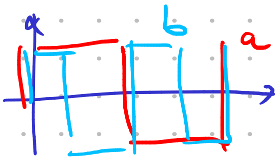
$+ a_1 \cdot a_2 \cos(\phi) \cos(2\phi) \cdots$

$\int_0^{2\pi} \cos(n\phi) \cdot \cos(m\phi)\,d\phi = 0$

$\Rightarrow \boxed{P = a_0^2 + \frac{a_1^2}{2} + \frac{a_2^2}{2} + \cdots + \frac{a_n^2}{2}}$

$x(t) = \cos(\omega t) + \sin(\omega t) + \cos(2\omega t)$

$\sqrt{2}\cos\left(\omega t - \frac{\pi}{4}\right)$



~~Увод и основни појмови.~~

~~**1.**[1] За следеће системе испитати да ли су стабилни у *BIBO* смислу, линеарни, временски инваријантни, са меморијом и каузални:~~

(а) $y(t)=\sum_{k=0}^{\infty} x(t-kT)$,

(б) $y(t)=\sqrt{2}x(t)$,

(в) $y(t)=tx(t-1)^2$,

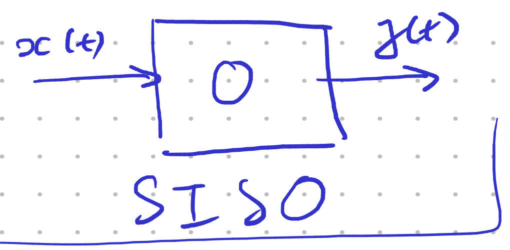
(г) $y(t)=\int_{\tau=-\infty}^{t} x(\tau)\sin(\tau)\,\mathrm{d}\tau$,

(д) $y(t)=\frac{\mathrm{d}x(t+1)}{\mathrm{d}t}$,

(ђ) $y(t)=te^{x(t)-t}$,

где је $y(t)=O\{x(t)\}$ одзив посматраног система.

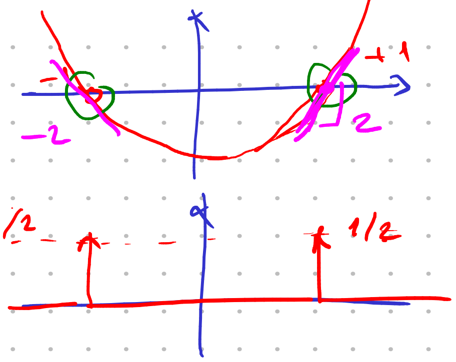
$y(t) = O\{x(t)\}$



$\delta(t^2-1)$

$\delta(at) = \frac{1}{|a|}\delta(t)$

$\frac{d}{dt}(t^2-1) = 2t$



Дираков делта импулс и Хевисајдова одскочна функција

1. Скицирати временске дијаграме следећих сигнала

(а) $x(t)=\delta(t^2-1)$; (б) $x(t)=\delta(\sin t)$; (в) $x(t)=\mathrm{rect}(t)\cdot\mathrm{tri}(t)\cdot\mathrm{u}(t)$,

$\boxed{x(t)=\delta(t^2-1)}$

$\delta(at) =$

$\int_{-\infty}^{\infty}\delta(at)\,dt = \int_{-\infty}^{\infty}\delta(at)\cdot\frac{d(at)}{|a|} = \frac{1}{|a|}$

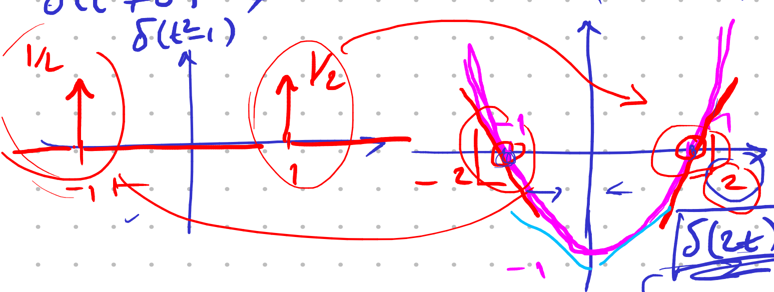
$\int_{-\infty}^{+\infty}\delta(t)\,dt \triangleq 1.$

$\delta(at) = \frac{1}{|a|}\delta(t).$

$\delta(t\neq 0) = 0$

$t^2-1 = (t-1)(t+1) \neq 0$

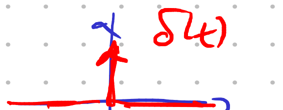
$t \notin \{-1,1\} \Rightarrow \delta(t^2-1) = 0.$



$\frac{d}{dt}(t^2-1) = 2t = 2$ @ $t=1$

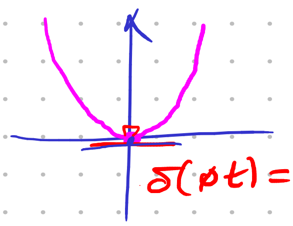
$\boxed{\delta(2t)} = \frac{1}{2}\delta(t)$

$\delta\left(\left|\frac{df}{dt}\right|_{t=t_0} t\right)$



$\delta(t) = \delta(-t)$

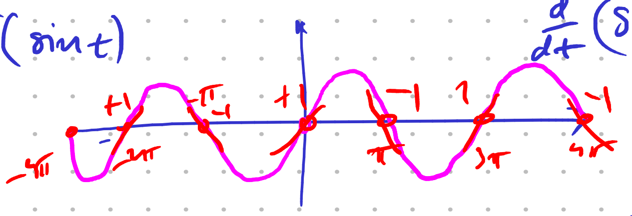
$\cancel{\delta(t^2)=}$



$\delta(0t) = \delta(0)$ Недеф. $= \frac{1}{0}\delta(n)$

(б) $\delta(\sin t)$

$\frac{d}{dt}(\sin t) = \cos t$

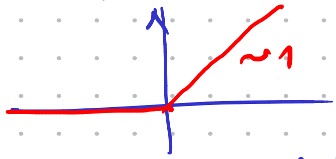


$|\cos t|$ @ $t = k\pi$

$\delta(\sin t) = \delta(t) + \delta(t-\pi) + \delta(t+\pi) + \dots = Ш_\pi(t).$

Zadatak 1.10.

$ramp(t) = t \cdot u(t)$



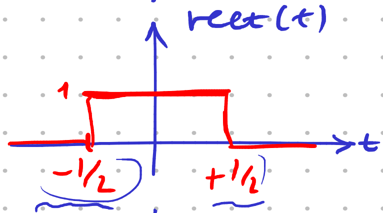
Signal je definisan izrazom: $x(t) = \mathrm{ramp}(t+3)\,\mathrm{rect}\left(\frac{t}{2}+1.5\right) + \mathrm{rect}\left(\frac{t}{4}\right)\left(1+\mathrm{e}^{-t}\,\mathrm{u}(t)\right)$.

a) Nacrtati zadati signal.

b) Nacrtati parni i neparni deo signala.

c) Nacrtati transformacije signala $g_1(t) = -3\,x\left(\frac{t}{2}-3\right)$, $g_2(t) = x(2t+2) - x(2t-2)$.

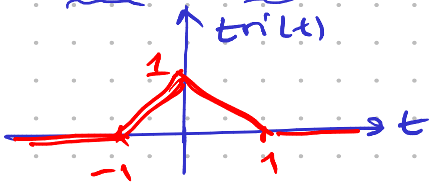
$rect(t)$



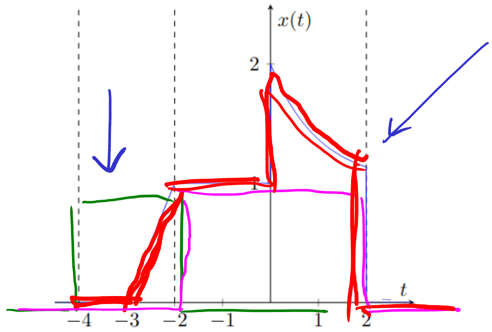
Rešenje:

a) Signal $x(t)$ je prikazan na slici 1.10.1.

$tri(t)$



$$\frac{x(t) \pm x(-t)}{2}$$



Slika 1.10.1.

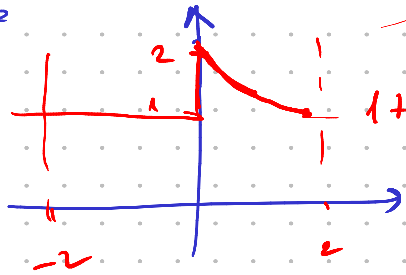
$rect\left(\underbrace{\frac{t}{2}+1,5}_{=\pm\frac{1}{2}}\right)$ $\qquad \frac{t}{2}+1,5 = \pm 0,5 \quad \therefore \quad \frac{t}{2} = -1,5 \pm 0,5 \quad /\cdot 2$

$t = -3 \pm 1 = \begin{cases} -4 \\ -2 \end{cases}$

$rect(t)$, $rect\left(\frac{t}{4}\right)$

$-\frac{1}{2} \div +\frac{1}{2}$ $\quad \boxed{-2 \div 2}$

$\left(1 + e^{-t}\,u(t)\right)$



$1+e^{-2} = 1+\frac{1}{e^2}$

$rect(t) \cdot tri(t)$