a) [6] $x(t)$ je realan periodični signali sa osnovnom učestanošću $\omega_0 = 1\text{rad/s}$. Ako su amplitudski i fazni spektar signala za vrednosti indeksa $k \geq 0$ dati izrazima $|X[k]| = k\nabla u[k-2] + \delta[k-4]$ i $\theta[k] = k\pi/4$, odrediti kompletan spektar signala $x(t)$. $|X[k]| = |X[-k]|$, $\theta[k] = -\theta[-k]$ $\quad X[k] = X^*[-k]$

$k \geq 0: |X[k]| = k\nabla u[k-2] + \delta[k-4] = k(u[k-2] - u[k-3]) + \delta[k-4] = k\delta[k-2] + \delta[k-4] = 2\delta[k-2] + \delta[k-4]$

Prema tome ispunjeno je da je $|X[k]| = |X[-k]|$ za svako k. Za fazni spektar vazi $\theta[k] = -\theta[-k]$

Konačno: $e^{j\theta[k]}$ $\quad 2\delta[k+2] + \delta[k+4]$

$$X[k] = 2\delta[k-2]e^{j2\pi/4} + \delta[k-4]e^{j4\pi/4} + 2\delta[-k-2]e^{-j2\pi/4} + \delta[-k-4]e^{-j4\pi/4} =$$

$$2\delta[k-2]e^{j\pi/2} + \delta[k-4]e^{j\pi} + 2\delta[k+2]e^{-j\pi/2} + \delta[k+4]e^{-j\pi} =$$

$$= 2j\delta[k-2] - \delta[k-4] - 2j\delta[k+2] - \delta[k+4]$$

$$X = \sum |X[k]|\, e^{j\theta[k]} \qquad P = \sum |X[k]|^2$$

b) [4] Kolika je srednja snaga a kolika energija signala $x(t)$? $P = 2^2 + 1 + 1 + 2^2 = 10$, $W = \infty$

c) [4] Odrediti izraze za koeficijente razvoja $A[k]$, i $B[k]$ u trigonometrijski Furijeov red neparnog dela signala $x(t)$

$A[k] = 0$, $B[k] = -2\operatorname{Im}\{X[k]\} = -4\delta[k-2] + 4\delta[k+2]$

d) [6] Ako je $y(t) = x(t)h(t)$ (množenje!), a $h(t) = 1 + \cos(3\omega_0 t) + \cos(5\omega_0 t)$, odrediti $Y[0]$. $\boxed{\mathbf{Y[0]=0}}$

e) [4] Ako se signal $x(t)$ razvije na periodi $T_1 = 2T_0$ za takav razvoj odrediti koeficijente razvoja $X_2[k]$

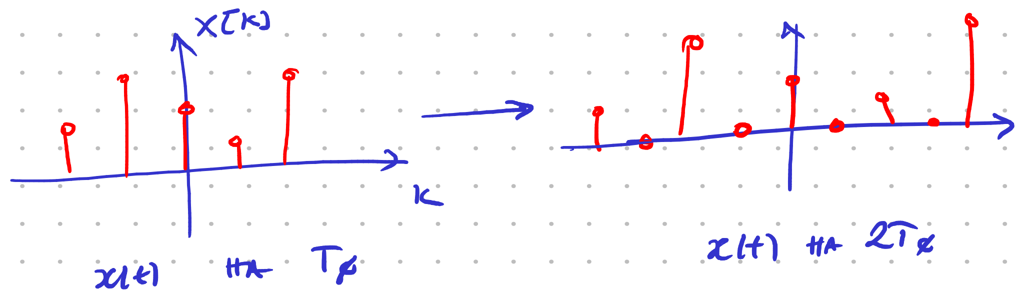
$$\boxed{X_2[k] = \begin{cases} X\left[\dfrac{k}{2}\right], k \text{ parno} \\ 0, k \quad \text{neparno} \end{cases}} \Rightarrow \text{ubacivanje nule izmedju svaka 2 stara odbirka}$$

$$X_2[k] = 2j\delta[k-4] - \delta[k-8] - 2j\delta[k+4] - \delta[k+8]$$

$$X[k] = \frac{A[k] - jB[k]}{2} \qquad \sim\cancel{\cos} \quad \sim\sin$$

c) $x(t)$ $\begin{cases} A[k] = 2\operatorname{Re}\{X[k]\} \\ B[k] = -2\operatorname{Im}\{X[k]\} \end{cases}$ $\quad$ Od $x_H(t)$ $\begin{cases} A[k] = \cancel{\phi} \\ B[k] = -2\operatorname{Im}\{X[k]\} \end{cases}$

d) $y(t) = x(t)\cdot h(t) \Rightarrow Y[k] = X[k] * H[k]$ $\quad \sin(n\omega_0 t)\cdot\sin(m\omega_0 t) = \phi$



$$X_m[k] = \begin{cases} X\left[\frac{k}{m}\right], m \mid k \\ \phi, m \nmid k \end{cases}$$

1. Dat je signal $x[n] = \cos\left(\frac{6\pi}{17}n + \pi/3\right)$. Odrediti osnovnu periodu i razviti signal u komplesan Furijeov red na osnovnoj periodi.

$$x[n] = x[n+N] \Rightarrow \cos\left(\frac{6\pi}{17}n + \frac{\pi}{3}\right) = \cos\left(\frac{6\pi}{17}(n+N) + \frac{\pi}{3}\right)$$

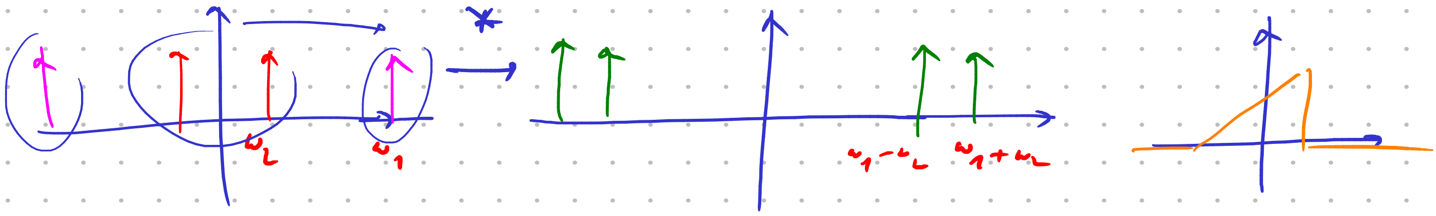
$$\Rightarrow \frac{\cancel{6}^{3}\cancel{\pi} N}{17} = \cancel{2}\cancel{\pi} m \Rightarrow N = \frac{17}{\cancel{3}}\cdot \cancel{m}, \quad m = 3 \Rightarrow \boxed{N = 17.}$$

$$\boxed{\Omega_0 = \frac{2\pi}{17}.} \qquad x[n] = \cos\left(3\Omega_0 n + \frac{\pi}{3}\right) = \frac{1}{2}e^{j(3\Omega_0 n + \frac{\pi}{3})} + \frac{1}{2}e^{-j(3\Omega_0 n + \frac{\pi}{3})}$$

$$x[n] = \underbrace{\boxed{\tfrac{1}{2}}\, e^{j3\Omega_0 n}\cdot \boxed{e^{j\frac{\pi}{3}}}}_{X[3],\ k=3} + \underbrace{\boxed{\tfrac{1}{2}}\, e^{-j3\Omega_0 n}\cdot \boxed{e^{-j\frac{\pi}{3}}}}_{X[-3],\ k=-3}$$

$x[n] = \sum_{m=0}^{N-1} X[m]\, e^{jm\Omega_0 n}$; $X[k] = \frac{1}{2} e^{j\frac{\pi}{3}}\delta[k-1] + \frac{1}{2} e^{-j\frac{\pi}{3}}\delta[k+1]$

$\sin(\omega_1 t)\cdot \sin(\omega_2 t) = \frac{1}{2}\left(\sin(\omega_1+\omega_2)t + \sin(\omega_1-\omega_2)t\right)$



$x(t) = x_I(t) + j x_Q(t)$

2. Dati su spektri dva signala periode 4: $F_s\{x[n]\} = X[k] = \{1,2,2,1\}$, i $F_s\{h[n]\} = H[k] = \{1,1,1,3\}$. Odrediti spektar signala $y[n] = x[n]\cdot h[n]$

$\mathcal{F}S\ x(t)$ $\mathcal{F}S\ x[n]$;

$Y[k] = X[k] \circledast H[k]$; $x[n] * y[n] = \sum_{m=-\infty}^{\infty} x[m]\, y[n-m]$ $\sum_{p+q=n} x[p]\cdot y[q]$

$$Y[k] = \sum_{n+m \equiv k\ (\mathrm{mod}\ N)} X[n]\cdot H[m]$$

	X[k]: 0	1	2	3
H[k]	1	2	2	1
0: 1	1	2	2	1
1: 1	1	2	2	1
2: 1	1	2	2	1
3: 3	3	6	6	3

$\boxed{Y[0] = 10}$; $\boxed{Y[2] = 8}$

$\boxed{Y[1] = 10}$; $\boxed{Y[3] = 8}$

2)

Osnovni period diskretnog periodičnog signala $x[n]$ je $N_0 = 8$, a koeficijenti razvoja signala $x[n]$ u kompleksni Furijeov red su $X[0]=1$, $X[1]=1.5-j0.5$, $X[-1]=1.5+j0.5$, $X[2]=j0.5$ i $X[-2]=-j0.5$. Odrediti signal $x[n]$.

$X[k] = X^*[-k]$

$X[3] = 0$

$x[n] = \sum_{\langle N_0\rangle} X[k]\, e^{jk\Omega_0 n} = \sum_{k=0}^{7} X[k]\, e^{jk\Omega_0 n}$

$X[k] = \frac{A[k] - jB[k]}{2}$

$A[1] = 3$ $B[1] = 1$

$A[2] = 0$ $B[2] = -1$

$A[0] = X[0] = 1$

$k>0$ $\begin{cases} A[k] = 2\,\mathrm{Re}\{X[k]\} \\ B[k] = -2\,\mathrm{Im}\{X[k]\} \end{cases}$

$$\boxed{x[n] = 1 + 3\cos(\Omega_0 n) + \sin(\Omega_0 n) - \sin(2\Omega_0 n).\quad \Omega_0 = \frac{\pi}{4}.}$$

$x[n] = A_0 + \sum A_k \cos k\Omega_0 n + \sum B_k \sin k\Omega_0 n$

$x(t) \in \mathbb{C}$

$A[k] = X[k] + X[-k]$

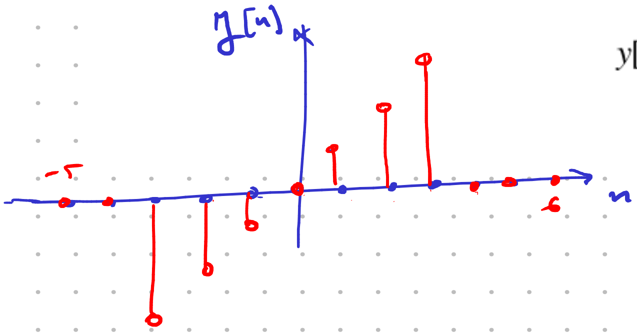
$B[k] = X[k] - X[-k]$

$$X[k] \to A e^{jk\Omega_0 n} = A\left(\cos(k\Omega_0 n) + j\sin(k\Omega_0 n)\right)$$

$$X[-k] \to B e^{-jk\Omega_0 n} = B\left(\cos(k\Omega_0 n) - j\sin(k\Omega_0 n)\right)$$

1)

Odrediti razvoj periodičnog diskretnog signala u kompleksni Furijeov red, ako je dat signal $y[n]$ za vrednosti promenljive n u toku jednog perioda:



$$y[n] = \begin{cases} n, & |n| \le 3 \\ 0, & 3 < n \le 6 \\ 0, & -5 \le n < -3 \end{cases}$$

$$N_0 = N_F = 12$$

$$\Omega_0 = \frac{2\pi}{12} = \frac{\pi}{6}.$$

$$Y[k] = \frac{1}{N} \sum_{n=\langle N_F \rangle} y[n]\, e^{-jk\Omega_0 n}$$

$$Y[0] = 0; \quad Y[1] = \frac{1}{12} \sum_{n=\langle N_F\rangle} y[n]\, e^{-j\boxed{1}\Omega_0 n} =$$

$$= \frac{1}{12}\Big(-3 e^{+j3\Omega_0} + (-2) e^{j2\Omega_0} + (-1) e^{j\Omega_0} + 0 + 1 e^{-j\Omega_0} + 2 e^{-j2\Omega_0} + 3 e^{-j3\Omega_0}\Big)$$

$$= \frac{1}{12}\Big(\underbrace{-3\left(e^{j3\Omega_0} - e^{-j3\Omega_0}\right)}_{} + \dots$$

$$-3 \cdot j2\sin(3\Omega_0) \quad \dots$$

$$Y[k] = \frac{1}{12} \sum_{n=\langle N_F\rangle} y[n]\, e^{-jk\Omega_0 n} =$$

$$e^{jx} - e^{-jx} = j2\sin x$$

$$= \frac{1}{12}\Big(-3 e^{+jk\Omega_0 3} + (-2) e^{jk2\Omega_0} + (-1) e^{jk\Omega_0} + \underline{+3 e^{-jk3\Omega_0}} \; \underline{+2 e^{-jk2\Omega_0}} + 1 e^{-jk\Omega_0}\Big)$$

$$\Rightarrow \quad \boxed{Y[k] = -\frac{j}{6}\left[\sin\left(\frac{k\pi}{6}\right) + 2\sin\left(\frac{k\pi}{3}\right) + 3\sin\left(\frac{k\pi}{2}\right)\right]}$$

РЕАЛАН !

2.[1] Одредити такав дискретан сигнал $x[n]$, са основним периодом $N = 6$ за који важе $\sum_{n=0}^{5} x[n] = 2$ и $\sum_{n=2}^{7} (-1)^n x[n] = 1$ такав да је његова средња снага минимална.

$$\Omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$\boxed{\sum_{n=0}^{5} x[n] = 2}$$

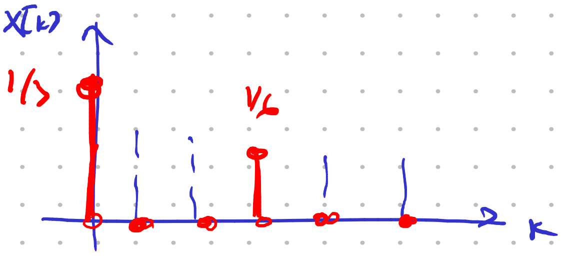
$$X[k] = \frac{1}{N} \sum_{n=\langle N_F \rangle} x[n] \left(e^{-jk\Omega_0}\right)^n$$

$$\sum_{n=2}^{7} (-1)^n x[n] = 1.$$

$$e^{j\pi} = e^{j\frac{\pi}{3}\cdot 3}$$

$$\boxed{X[3] = \frac{1}{6}} \sum_{n=2}^{7} x[n] \cdot \Big(\underbrace{e^{-j3\frac{\pi}{3}}}_{-1}\Big)^n = \frac{1}{6}$$

$$X[0] = \frac{1}{6} \sum_{n=0}^{5} x[n] \Rightarrow \boxed{X[0] = \frac{1}{3}}$$



$$P = \sum_{k=\langle N_F \rangle} |X[k]|^2 \leftarrow \min$$

$$P = \frac{1}{9} + \frac{1}{36}.$$

$$\boxed{x[n] = \frac{1}{3} + \frac{1}{3}\cos(\pi n)}$$

$$X[3] = 1/6 \,, \quad X[-3] = \frac{1}{6}$$

$$A[3] = 2\,\mathrm{Re}\, X[3] = \frac{1}{3}$$