LTI SISTEMI

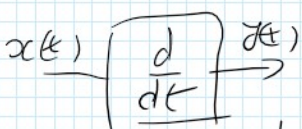
1. KONTINUALNI
2. DISKRETNI

$O\{x(t)\} = y(t)$

$O\{x[n]\} = y[n]$

SISO

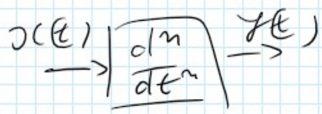
MIMO

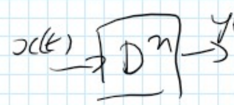


$y(t) = \frac{d}{dt} x(t)$

$\boxed{\frac{d}{dt} = D}$

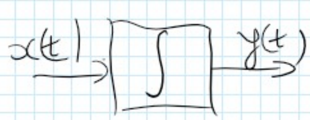
$y(t) = \frac{d}{dt} x(t)$



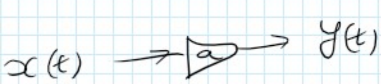


$y(t) = \frac{d^n}{dt^n} x(t)$

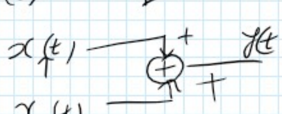
$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$



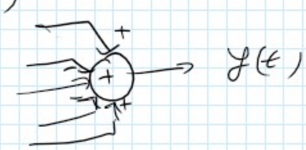
$y(t) = \underbrace{\iint\int}_{n} x(\tau)\,d\tau$

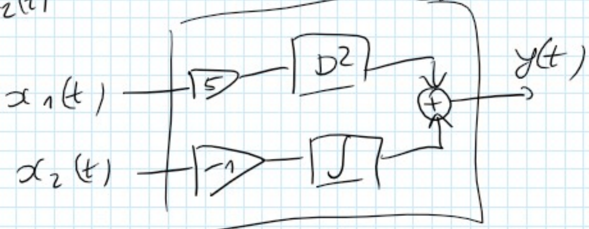


$y(t) = a\,x(t)$

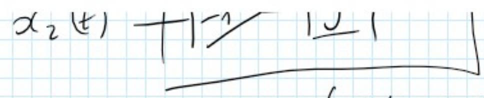


$y(t) = x_1(t) + x_2(t)$





$O\{x_1(t) + x_2(t)\}$ … $y(t)$



$$O\{x_1(t), x_2(t)\} = y(t)$$

(1) ШЕМАТСКИ ОПИС } KONTINUALNI | D, $\int$, (+), ▷, I
DISKRETНИ | D, E, ∇, Δ, I, (+), ▷

(2) LINEARНA DIFERENCIJALNA JEDNAČINA SA KK

$D \equiv \frac{d}{dt}$ $\quad a_n D^n y(t) + a_{n-1} D^{n-1} y(t) + \dots a_0 D^0 y(t) =$

$$\underbrace{b_m D^m x(t) + b_{m-1} D^{m-1} x(t) + \dots b_0 D^0 x(t)}$$

$$\boxed{P_n(D)\, y(t) = Q_m(D)\, x(t)}$$

$P(D) = a_n D^n + a_{n-1} D^{n-1} \dots + a_0$ } TEK!

$Q(D) = b_m D^m + b_{m-1} D^{m-1} + \dots b_0$

$$\boxed{n \geq m}$$

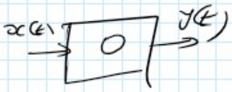
DIGRESIJA: OPIS PREKO DIJAGRAMA

(1) BLOK DIJAGRAMI — SIGNAL FLOW OPIS

a) SAMO VREDNOSTI „POTENCIJALA"

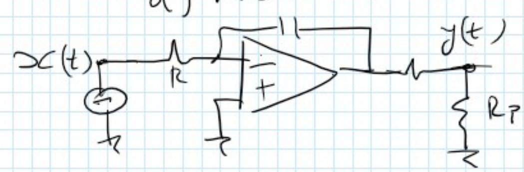
б) BLOKOVI SU UNIDIREKCIONI

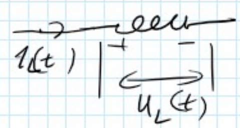
в) PROCESIRAJU SE „ČISTE INFORMACIJE



(2) ELEKTRIČNA ŠEMA — KONZERVATIVNI OPIS

a) VAŽE ZAKONI ODRŽANJA ENERGIJE!





$f(i_L(t), u_L(t)) = 0$

$H(D)$
$H(j\omega)$
$H(s)$

$$P(D)\, y(t) = Q(D)\, x(t)$$

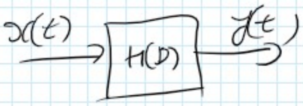
$$y(t) = \frac{Q(D)}{P(D)} x(t) = H(D)\, x(t)$$

$$H(D) = \frac{Q(D)}{P(D)}$$

↑ SISTEM OPERATOR POTPUNO OPISUJE SISTEM

— OPIS U VREMENSKOM DOMENU $\quad O\{x(t)\} = y(t)$

$x(t) \longrightarrow y(t)$



$H(D)\, x(t) = y(t)$

— OBLAST OD INTERESA $\quad t_0 < t < t_1 \quad t_0 = 0$

$0 < t < \infty$ $\quad$ | $y(t)$ | $y(t - 10s)$ |

POBUDA: $x(t) = g(t) \cdot u(t) \longrightarrow x(t)$ JE KAUZALNO

$y(t)$ JE PO PRIRODI KAUZALNO

POBUDA: $x(t) = g(t) \cdot u(t) \longrightarrow x(t)$ JE KAUZALNO

$y(t)$ JE PO PRIRODI KAUZALNO

UNUTRAŠNJA ENERGIJA : POČETNI USLOVI OD PRE POČETKA DEJZOVANJA POBUDE!

$t = 0^-$ $\quad t = 0 - \varepsilon$, $\varepsilon \to 0$, $\varepsilon > 0$

$$\underbrace{y(t=0^-),\ y'(t=0^-) \ldots}_{\text{PREINICIJALNI USLOVI, } n \text{ UKUPNO}}$$

POTREBNI SU POSTINICIJALNI USLOVI: USLOVI OD $t = 0^+$; $t = 0 + \varepsilon$, $\varepsilon \to 0$, $\varepsilon > 0$

$$\underbrace{y(t=0^+),\ y'(t=0^+) \ldots}_{n \text{ USLOVA UKUPNO}}$$

$$RHS = \underbrace{Q(D)\, x(t)}_{\text{MOGUĆE JE DA SE POJAVE } \delta(t),\ \delta'(t),\ \delta''(t) \ldots} \qquad x(t) = g(t)\, u(t)$$

PREINICIJALNI POČ USLOVI $\neq$ POSTINICIJALNI POČ USLOVI

SPEC SLUČAJ $\quad Q(D) = k$

$x(t) = g(t) \cdot u(t)$

$RHS = k \cdot g(t) \cdot u(t)$

$y(0^+) = y(0^-)$

$y'(0^+) = y'(0^-)$

INICIJALNI POČETNI USLOVI

$\underline{y(t)}$

NALAŽENJE ODZIVA SISTEMA REŠAVANJEM DIFERENCIJALNE JEDNAČINE

$\boxed{x(t) = e^{at} \cdot u(t)}$ $\quad \boxed{x(t) = A\cos(\omega t + \theta)\, u(t)}$

$x(t) = R(t) \cdot e^{at}\, u(t)$

$= R(t) \cdot \cos(\omega t + \theta) \cdot u(t)$

$= R(t) \cdot e^{at} \cos(\omega t + \theta)\, u(t)$

$$\underbrace{Q(D)\, x(t)}_{RHS} \qquad (D+1)\, g(t) \cdot u(t) = \underbrace{D \cdot g(t)\, u(t)}_{y_1(t)} + \underbrace{g(t)\, u(t)}_{y_2(t)}$$

$a = \alpha$

$a = j\omega$

$a = \sigma + j\omega$

$R(t) = t^3 + 2t^2 + t + 8$

$D\, g(t)\, u(t) = g'(t)\, u(t) + g(t)\, \delta(t)$

$\boxed{\text{AKO JE } g(0) = 0 \Rightarrow g(t)\, \delta(t) = g(0)\, \delta(t) = 0}$

— PRIMENA OPERACIONOG RAČUNA

① $P(\lambda) = 0$

$\alpha_1, \alpha_2, \ldots$

— REALNE — PRVOG REDA / VIŠEG REDA

— KOMPLEKSNE — PRVOG REDA / VIŠEG REDA

$\Longrightarrow$ STABILNOST SISTEMA:

(1) DOVOLJAN USLOV STABILNOSTI

$$\boxed{Re\{\alpha_k\} < 0} \text{ ZA SVAKO } k$$

(2) MARGINALNA STABILNOST

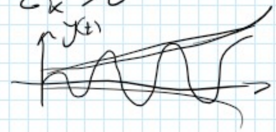
$R(t)\, e^{\alpha_k t} = R(t)\, e^{(\sigma_k + j\omega_k) t} =$

$= R(t)\, e^{\sigma_k t}\, (e^{j\omega_k t})$

$$\boxed{Re\{\alpha_k\} < 0 \text{ ZA SVAKO K}}$$

$$= R(t)\, e^{\sigma_k t} (e^{j\omega_k t})$$

$\sigma_k > 0$



$e^{\sigma_k \cdot t}$

e^{3t}

(2) MARGINALNA STABILNOST

$Re\{\alpha_{k=k_0}\} = 0$

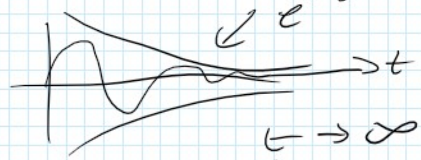
$$y(t) = \ldots + R(t) \cdot e^{0 \cdot t} e^{j\omega_k t} \ldots$$

— $R(t) = (t+1)\, e^{j\omega_k t}$

— $R(t) = K$

$\sigma_k < 0$

e^{-3t}



$t \to \infty$

$y(t) \to 0$

$$y(t) = y_h(t) + y_p(t) \quad \text{← SPEC. OBLIK}$$

$$\underline{P(D)\, y_h(t) = 0} \qquad \underline{y_h(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \ldots}$$

ZA SLUČAJ REALNIH I RAZLIČITIH KORENOVA

$$P(D)\, y_p(t) = \overbrace{Q(D)\, x(t)}^{\text{NEMA } \delta(t), \delta'(t) \ldots}$$

$$y(t) = \frac{Q(D)}{P(D)} x(t) = H(D)\, x(t)$$

$$x(t): \begin{cases} c \cdot e^{at} & a \neq \lambda_k \text{ ZA BILO KOJE K} \\ c \cdot 1 = c = c \cdot e^{0 \cdot t} \end{cases}$$

$a = \lambda_k$
POBUDA JE U REZONANSI SA SISTEMOM!

$$y_p(t) = c \cdot H(a)\, e^{at}$$

$$y_p(t) = c \cdot H(0) \cdot e^{0 \cdot t} = c \cdot H(0)$$

$$y(t) = y_h(t) + y_p(t) = \underbrace{c_1 e^{\lambda_1 t} + \ldots c_n e^{\lambda_n t}}_{n \text{ NEODREĐENIH KOEFICIJENATA}} + c \cdot H(a) \cdot e^{at}$$

$\Rightarrow$ ODREĐUJU SE SA n POČETNIH USLOVA

$y(t)$ ZA $0 < t < t_1 < \infty$

$t = 0^+$

$y(0^+), y'(0^+) \ldots$

$y(t); \quad x(t) = g(t) \cdot u(t)$

$0 < 0^+ < t_1$

ZA $t \geq 0^+$ $\quad x(t) = g(t)$

$\boxed{y(t) \text{ NEPREKIDNO U } 0}$
$x(t)$ NEPREKIDNO ZA $0 < t < t_1$
} DOVOLJNO n POMOĆNIH USLOVA

$y(t_0), y'(t_0) \ldots$

$0 < t_0 < t_1$

$$\boxed{0 < 0^+ < t_1}$$

$0^-, 0$

$y(t) \quad x(t)$ ZA $t > 0 \quad x(t) = A \cos(\omega t + \theta) \quad x(t) = A \cos(3t + \theta)$

$0^-, 0$

$y_p(t)$, $x(t)$ ZA $t>0$ $\quad x(t) = A \begin{cases} \cos(\omega t+\theta) \\ \sin(\omega t+\varphi) \end{cases}$ $\quad x(t) = A\cos(3t+\theta)$

$\uparrow$

$\omega = 3$

$$y_p(t) = \frac{Q(D)}{P(D)}x(t) = Q(D)\frac{R(D)}{P(D)R(D)}x(t)$$

$$\longrightarrow x(t) = A\left(\frac{e^{j(3t+\theta)} + e^{-j(3t+\theta)}}{2}\right) \text{ ZA } t>0$$

$$= \frac{Q(D)\cdot R(D)}{P_1(D^2)}x(t) =$$

$$\longrightarrow x(t) = A\left(\frac{e^{j(\cdot)} + e^{j(\cdot)}}{2}\right)\cdot u(t)$$

ZA SVAKO t

$$= Q(D)\cdot R(D)\cdot \frac{A\cos(\omega t+\theta)}{P_1(-\omega^2)}$$

$$\frac{A}{P_1(-\omega^2)} = B$$

$$= \underbrace{Q(D)\cdot R(D)}\cdot B\cos(\omega t+\theta)$$

$$(D^2+2D+1)\cdot B\cos(\omega t+\theta) = B\left(\cos''(\omega t+\theta) + 2\cos'(\omega t+\theta) + \cos(\omega t+\theta)\right)$$

KADA JE SISTEM STABILAN:

$$y_h(t) = \sum_{k=1}^{n} c_k R_k(t) e^{\lambda_k \cdot t}$$

$$y_h(t) = \sum c_k e^{\lambda_k t} \quad \mathrm{Re}\{\lambda_k\}<0$$

$t\to\infty \quad y_h(t)\to 0$

$\uparrow$ PRELAZNI ODZIV

$$x(t): c\cdot e^{at,\, a=0} = c \longrightarrow \text{USTALJENI DC REŽIM}$$

$+$ NEMA REZONANSE

$$\begin{array}{c} 0 \\ \shortparallel \\ a \neq \lambda_k \\ j\omega \neq \lambda_k \end{array}$$

$: A\cdot\cos(\omega t+\theta) \longrightarrow$ USTALJENI PROSTOPERIODIČNI REŽIM

$x(t) = c\cdot e^{at}$ ZA $t>0$

$x(t) = c\cdot e^{at}\cdot u(t)$

$a=0$

$x(t) = c$ ZA $t>0$

ZA $t<t_1$

$$\boxed{x(t) = c\cdot u(t)}$$

$\uparrow$

$$\boxed{y_p(t) = H(0)\cdot c}$$

IMPULSNI ODZIV I KONVOLUCIJA

$$x(t) \to \boxed{O} \to y(t) \qquad x(t) \to \boxed{H(D)} \to y(t)$$

$$\delta(t) \to \boxed{} \to h(t)$$

$$x(t) \to \boxed{h(t)} \to y(t)$$

SISTEM BEZ UNUTRAŠNJE ENERGIJE

(SA NULTIM PREDINICIJALNIM USLOVIMA)

$$y(t) = x(t) * h(t) = h(t) * x(t)$$

$\uparrow$

KONVOLUCIJA

$$x(t) * h(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)\,d\tau$$

(1) $\quad x(t) \to \boxed{h(t)} \to y(t) \qquad h(t) \to \boxed{x(t)} \to y(t)$

$y(t) = x(t) * h(t)$

$y(t) = h(t) * x(t)$

$-\infty$

(1) $x(t) \to \boxed{h(t)} \to y(t)$ $\qquad h(t) \to \boxed{x(t)} \to y(t)$ $\qquad y(t) = x(t) * h(t)$

$y(t) = h(t) * x(t)$

(2) $\delta(t) \to \boxed{h(t)} \to h(t)$ $\qquad \delta(t) \to \boxed{x(t)} \to x(t)$

$\delta(t) * x(t) = x(t)$

(3) $x_1(t) * (x_2(t) * x_3(t)) = (x_1(t) * x_2(t)) * x_3(t)$

$\to \boxed{h_1(t)} \to \left(\boxed{h_2(t)} \to \boxed{h_3(t)}\right) \to$ $\qquad \to \left(\boxed{h_1} \to \boxed{h_2}\right) \to \boxed{h_3} \to$

(4) $x_1(t) * (x_2(t) + x_3(t)) = x_1(t) * x_2(t) + x_1(t) * x_3(t)$

(5) $y(at) \neq x(at) * h(at)$

$y(at) = |a|\, x(at) * h(at)$

$\underbrace{\to \boxed{h_1(t)} \to \boxed{h_2(t)} \to}_{h_1 * h_2}$ $\qquad \underbrace{\to \boxed{h_2(t)} \to \boxed{h_1(t)} \to}_{h_2 * h_1}$

$x(t) \to \boxed{h(t)} \to y(t)$ $\qquad x(t) \to \boxed{D} \to \boxed{h(t)} \to y_1(t)$

$\frac{d}{dt} x(t) \to \boxed{h(t)} \to \overset{?}{y_1(t)}$ $\qquad x(t) \to \boxed{h(t)} \xrightarrow{y(t)} \boxed{D} \to y_1(t)$ $\qquad y_1(t) = Dy(t)$

$x(t) \to \boxed{h(t)} \to y(t)$

$Dx(t) \to \boxed{h(t)} \to Dy(t)$

$\delta(t-t_0) \to \boxed{h(t)} \to h(t-t_0)$

$x(t-t_0) \to \boxed{h(t)} \to y(t-t_0)$

$x(t) \to \boxed{h(t)} \to y(t)$

$u(\tau) = 0$
ZA $\tau < 0$

$u(t-\tau) = 0$
ZA $t-\tau < 0$
$t < \tau$
$\tau > t$

$u(t-\tau) = 0$
ZA $\tau > t$

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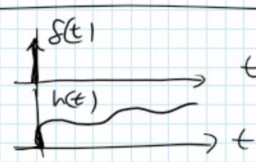
(1) KAUZALNI SISTEMI
(2) KAUZALNA POBUDA



$h(t) = h_1(t)\, u(t)$

$x(t) = x_1(t) \cdot u(t)$

$$h(t) * x(t) = \int_{-\infty}^{+\infty} x(\tau) h(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} \underbrace{x_1(\tau) u(\tau)\, h_1(t-\tau) u(t-\tau)}\, d\tau$$

$$= \left(\int_0^t x_1(\tau)\, h_1(t-\tau)\, d\tau \right) u(t)$$

$$= \left(\int_0 x_1(\tau)\, h_1(t-\tau)\, d\tau \right) u(t)$$

$x(t)\,,\ h(t)$ $\qquad h(t) * x(t) = x(t) * h(t) = \left(\int_{0^-}^{t} x(\tau)\, h(t-\tau)\, d\tau \right) \cdot u(t)$

$x(t) = e^{at} u(t)$

$h(t) = e^{bt} u(t)$ $\qquad \boxed{x(t) * h(t) = \frac{1}{a-b}\left(e^{at} - e^{bt}\right) u(t)}$

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① $h(t) \leftarrow$ ODZIV SISTEMA BEZ ENERGIJE } NULTI PREINICIJALNI USLOV

$\downarrow$

② $y(t) = y_{ZS}(t) + y_{ZI}(t)$

$\uparrow$ ODZIV SISTEMA NA UNUTRAŠNJU ENERGIJU (NA PREINICIJALNE USLOVE)

$y_{zs}(t) = h(t) * x(t)$

$y_{zi}(t) \Leftrightarrow$ $\boxed{P(D)\, y_{zi}(t) = 0}$ — NEODREĐENI KOEFICIJENTI SE DOBIJAJU NA OSNOVU PREINICIJALNIH USLOVA!

ODMAH SE RADI ZAMENA

$y_{zs} \leftarrow$ PRINUDNI ODZIV $\rightarrow y_{PR}(t)$ — SAMO NA POBUDU

$y_{zi} \leftarrow$ SOPSTVENI ODZIV $\rightarrow y_{SP}(t)$ — SAMO NA POČETNE USLOVE

$h(t)$ $\qquad \delta(t) \rightarrow \boxed{h(t)} \rightarrow h(t)$

$P(D)\, h(t) = Q(D)\,\delta(t)$

PREINICIJALNI USLOVI = 0

$\Downarrow$

POSTINICIJALNI USLOVI

$\boxed{P(D)\, h(t) = 0 \text{ ZA } t > 0 \text{ SA ODREĐENIM } h(0^+)\ h'(0^+) \ldots}$

$Q(D)\cdot\delta(t) \rightarrow \boxed{\frac{1}{P(D)}} \rightarrow h(t)$ $\qquad$ POMOĆNI IMPULSNI ODZIV

$\delta(t) \rightarrow \boxed{\frac{1}{P(D)}} \rightarrow h_1(t)$ $\qquad h(t) = Q(D)\cdot h_1(t)$

$\downarrow$

$P(D)\, S(t) = u(t)$ $\qquad u(t) = 1 = c\, e^{0\cdot t}$ ZA $t > 0$

$P(D)\, h_1(t) = \delta(t)$

$S(t) = \underbrace{c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \ldots}_{S_h(t)} + \frac{1}{P(0)} \cdot e^{0\cdot t}$ $\quad \rightarrow 1$

$u(t) \rightarrow \boxed{h_1(t)} \rightarrow S(t)$

$S(0^+) = S'(0^+) = \ldots\ 0$

$\underbrace{D\cdot u(t)}_{\delta(t)} \rightarrow \boxed{h_1(t)} \rightarrow D\cdot S(t) = h_1(t)$

$h_1(t) = S'(t)$

$h(t) = Q(D)\, h_1(t)$

$\delta(t)$ $\qquad h_1'(t)$ $\qquad \underline{h(t) = Q(D)\,h_1(t)}$

$$s(t) = s_g(t) \cdot u(t)$$

$$s'(t) = s_g'(t)\,u(t) + s_g(t) \cdot \delta(t) = h_1(t)$$

IMA PRAVILO ALI RADIĆEMO POSTUPNO!

$n = m \qquad h(t) = k \cdot \delta(t) + (\quad)$

$$\boxed{n > m \qquad h(t) = (\quad \underbrace{\cancel{\delta}}_{\text{NEMA } \delta(t)})}$$

$s'(t) = s_g(t) = h_1(t)$

$h(t) = (Q(D)\,h_1(t))\,u(t)$