Dynamic Models of Converters in the Discontinuous Conduction Mode

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1 Preliminaries

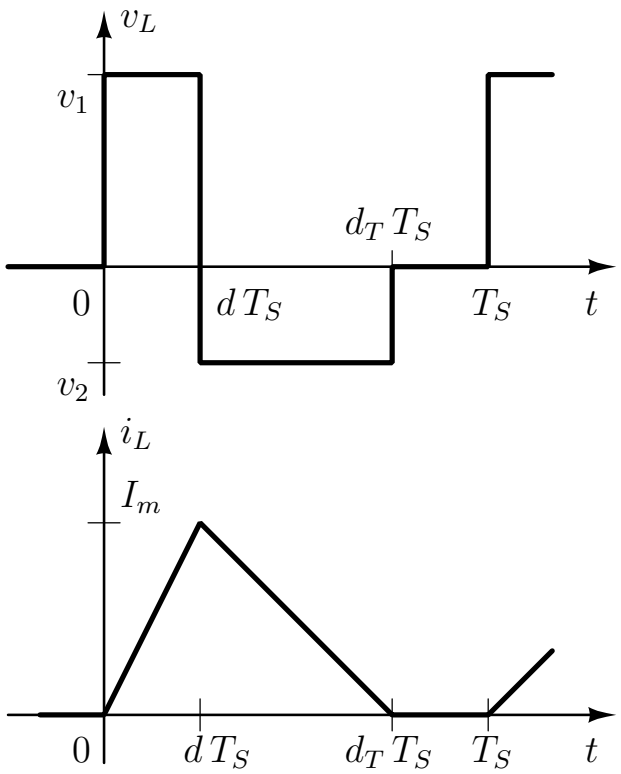

Figure 1: Waveforms of v_L and i_L, discontinuous conduction mode; $d_T = d + d_2$.

Table 1: Inductor Voltages and Average Current to the Capacitor and the Load

	buck	boost	buck-boost
v_1	$v_{IN} - v_{OUT}$	v_{IN}	v_{IN}
v_2	$-v_{OUT}$	$v_{IN} - v_{OUT}$	v_{OUT}
$\langle i_{CL} \rangle$	$\frac{1}{2}\,(d + d_2)\,I_m$	$\frac{1}{2}\,d_2\,I_m$	$-\frac{1}{2}\,d_2\,I_m$

In pulse-width modulation

$$I_m = \frac{v_1}{L}\,d\,T_S \tag{1}$$

while in the current mode control

$$d = \frac{f_S\,L\,I_m}{v_1}. \tag{2}$$

In both of the control methods

$$d_2 = -\frac{v_1}{v_2}\, d. \tag{3}$$

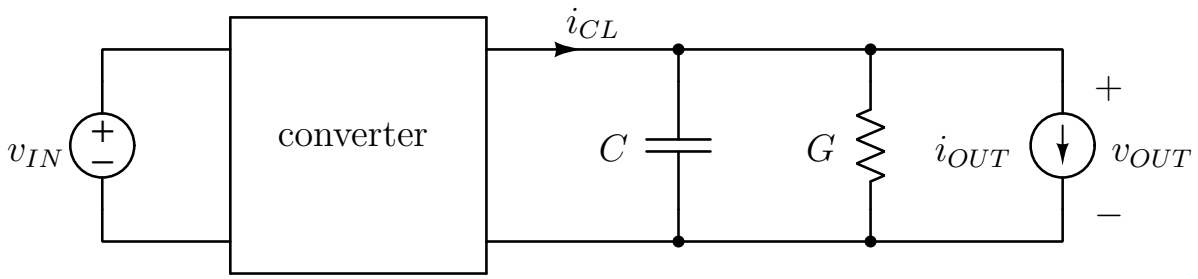

Figure 2: Assumed converter structure.

2 Pulse-Width Modulation

2.1 Nonlinear Dynamic Model

$$\frac{d\,v_{OUT}}{dt} = \frac{1}{C}\,i_{CL} - \frac{G}{C}\,v_{OUT} - \frac{1}{C}\,i_{OUT} \tag{4}$$

$$G_0 \triangleq \frac{1}{2\,f_S\,L} \tag{5}$$

Table 2: Pulse-Width Modulation, Nonlinear Dynamic Model

converter	i_{CL}	nonlinear dynamic model
buck	$G_0\,d^2\,v_{IN}\,\dfrac{v_{IN}-v_{OUT}}{v_{OUT}}$	$\dfrac{d\,v_{OUT}}{dt} = \dfrac{G_0}{C}\,d^2\,v_{IN}\,\dfrac{v_{IN}-v_{OUT}}{v_{OUT}} - \dfrac{G}{C}\,v_{OUT} - \dfrac{1}{C}\,i_{OUT}$
boost	$G_0\,d^2\,\dfrac{v_{IN}^2}{v_{OUT}-v_{IN}}$	$\dfrac{d\,v_{OUT}}{dt} = \dfrac{G_0}{C}\,d^2\,\dfrac{v_{IN}^2}{v_{OUT}-v_{IN}} - \dfrac{G}{C}\,v_{OUT} - \dfrac{1}{C}\,i_{OUT}$
buck-boost	$G_0\,d^2\,\dfrac{v_{IN}^2}{v_{OUT}}$	$\dfrac{d\,v_{OUT}}{dt} = \dfrac{G_0}{C}\,d^2\,\dfrac{v_{IN}^2}{v_{OUT}} - \dfrac{G}{C}\,v_{OUT} - \dfrac{1}{C}\,i_{OUT}$

2.2 Steady State Equations

Table 3: Pulse-Width Modulation, Steady State Equations

converter	steady state equations
buck	$G_0\,D_0^2\,V_{IN}\,\dfrac{V_{IN}-V_{OUT}}{V_{OUT}} - G\,V_{OUT} - I_{OUT} = 0$
boost	$G_0\,D_0^2\,\dfrac{V_{IN}^2}{V_{OUT}-V_{IN}} - G\,V_{OUT} - I_{OUT} = 0$
buck-boost	$G_0\,D_0^2\,\dfrac{V_{IN}^2}{V_{OUT}} - G\,V_{OUT} - I_{OUT} = 0$

2.3 Linearized Model

$$\frac{d\,\widehat{v}_{OUT}}{dt} = a_1\,\widehat{v}_{OUT} + a_2\,\widehat{v}_{IN} + a_3\,\widehat{i}_{OUT} + a_4\,\widehat{d} \tag{6}$$

Table 4: Pulse-Width Modulation, Small-Signal Parameters

converter	buck	boost	buck-boost
a_1	$-\dfrac{G_0}{C}\,D_0^2\,\dfrac{V_{IN}^2}{V_{OUT}^2} - \dfrac{G}{C}$	$-\dfrac{G_0}{C}\,D_0^2\,\dfrac{V_{IN}^2}{(V_{OUT}-V_{IN})^2} - \dfrac{G}{C}$	$-\dfrac{G_0}{C}\,D_0^2\,\dfrac{V_{IN}^2}{V_{OUT}^2} - \dfrac{G}{C}$
a_2	$\dfrac{G_0}{C}\,D_0^2\left(\dfrac{2\,V_{IN}}{V_{OUT}} - 1\right)$	$\dfrac{G_0}{C}\,D_0^2\,\dfrac{V_{IN}\,(2\,V_{OUT}-V_{IN})}{(V_{IN}-V_{OUT})^2}$	$2\,\dfrac{G_0}{C}\,D_0^2\,\dfrac{V_{IN}}{V_{OUT}}$
a_3	$-\dfrac{1}{C}$	$-\dfrac{1}{C}$	$-\dfrac{1}{C}$
a_4	$2\,\dfrac{G_0}{C}\,D_0\,V_{IN}\,\dfrac{V_{IN}-V_{OUT}}{V_{OUT}}$	$2\,\dfrac{G_0}{C}\,D_0\,\dfrac{V_{IN}^2}{V_{OUT}-V_{IN}}$	$2\,\dfrac{G_0}{C}\,D_0\,\dfrac{V_{IN}^2}{V_{OUT}}$

3 Current Mode Control

3.1 Nonlinear Dynamic Model

$$C \frac{d\, v_{OUT}}{dt} = i_{CL} - G\, v_{OUT} - i_{OUT} \tag{7}$$

Table 5: Current Mode Control, Nonlinear Dynamic Model

converter	i_{CL}
buck	$\frac{1}{2} \frac{v_{IN}}{v_{OUT}\,(v_{IN} - v_{OUT})} f_S\, L\, I_m^2$
boost	$\frac{1}{2} \frac{1}{v_{OUT} - v_{IN}} f_S\, L\, I_m^2$
buck-boost	$\frac{1}{2} \frac{1}{v_{OUT}} f_S\, L\, I_m^2$

3.2 Steady State Equations

Table 6: Current Mode Control, Steady State Equations

converter	steady state equation
buck	$\frac{1}{2} \frac{V_{IN}}{V_{OUT}\,(V_{IN} - V_{OUT})} f_S\, L\, I_m^2 - G\, V_{OUT} - I_{OUT} = 0$
boost	$\frac{1}{2} \frac{1}{V_{OUT} - V_{IN}} f_S\, L\, I_m^2 - G\, V_{OUT} - I_{OUT} = 0$
buck-boost	$\frac{1}{2} \frac{1}{V_{OUT}} f_S\, L\, I_m^2 - G\, V_{OUT} - I_{OUT} = 0$

3.3 Linearized Model

$$\frac{d\, \widehat{v}_{OUT}}{dt} = a_1\, \widehat{v}_{OUT} + a_2\, \widehat{v}_{IN} + a_3\, \widehat{i}_{OUT} + a_4\, \widehat{I}_m \tag{8}$$

Table 7: Current Mode Control, Small-Signal Parameters, Part 1

converter	a_1	a_2
buck	$\frac{f_S\, L}{2\, C} \frac{V_{IN}\,(2\, V_{OUT} - V_{IN})}{V_{OUT}^2\,(V_{IN} - V_{OUT})^2} I_m^2 - \frac{G}{C}$	$-\frac{f_S\, L}{2\, C} \frac{1}{(V_{IN} - V_{OUT})^2} I_m^2$
boost	$-\frac{f_S\, L}{2\, C} \frac{1}{(V_{OUT} - V_{IN})^2} I_m^2 - \frac{G}{C}$	$\frac{f_S\, L}{2\, C} \frac{1}{(V_{OUT} - V_{IN})^2} I_m^2$
buck-boost	$-\frac{f_S\, L}{2\, C} \frac{1}{V_{OUT}^2} I_m^2 - \frac{G}{C}$	0

Table 8: Current Mode Control, Small-Signal Parameters, Part 2

converter	a_3	a_4
buck	$-\frac{1}{C}$	$\frac{f_S L}{C} \frac{V_{IN}}{V_{OUT}\,(V_{IN} - V_{OUT})} I_m$
boost	$-\frac{1}{C}$	$\frac{f_S L}{C} \frac{I_m}{V_{OUT} - V_{IN}}$
buck-boost	$-\frac{1}{C}$	$\frac{f_S L}{C} \frac{I_m}{V_{OUT}}$