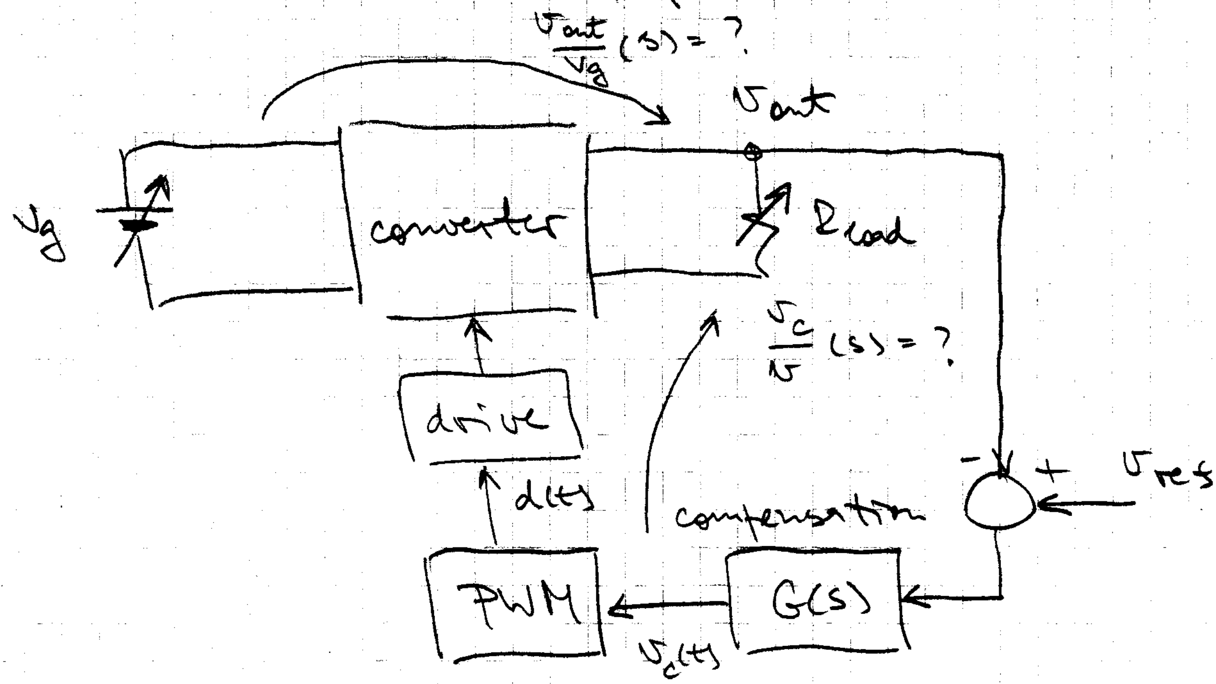
* Управљање DC-DC конверторима

- Типична апликација

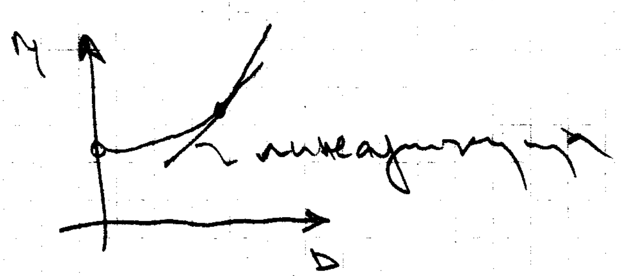


1 transient overshoot
2 setting time
3 steady-state regulation

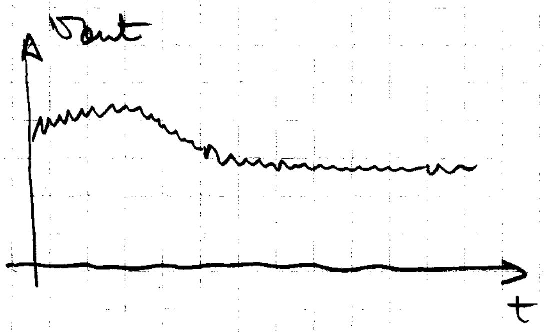
- Систем: нелинеарни / нестационарни
- У пракси - симулација - не важи за design, потребна анализа
- Аналитички проблеми:

1) нелинеарност

boost, $M(D) = \frac{1}{1-D}$ — ово је steady-state



2) ripple ([illegible])



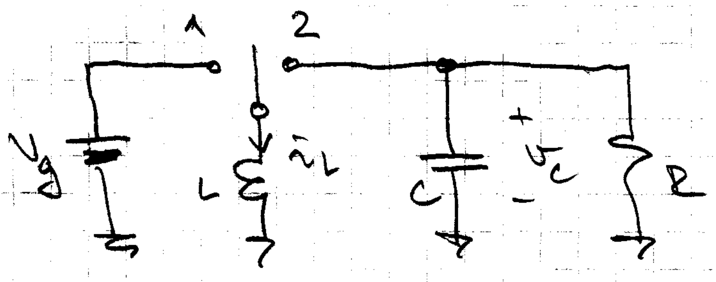
решење - neglect ripple

Процедура:

1) усредњавање

2) линеаризација

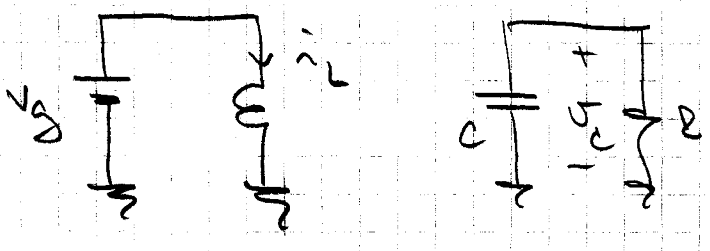
Пример: Buck-boost конвертор



T_s - switching period

D - duty ratio

position 1

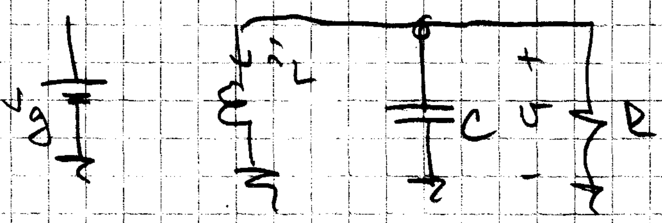


DT_s

$$v_L = L\frac{di_L}{dt} = V_g$$

$$i_C = C\frac{dv_C}{dt} = -\frac{v_C}{R}$$

position 2



$(1-D)T_s$

$$v_L = L\frac{di_L}{dt} = v_c$$

$$i_c = C\frac{dv_c}{dt} = -i_L - \frac{v_c}{R}$$

υπολογίζουμε $\hat{i}_L(T_s)$ κ $v_c(T_s)$ σαν φ-ση ως $\hat{i}_L(0)$, $v_c(0)$ κ D

$$\hat{i}_L(DT_s) = \hat{i}_L(0) + DT_s\frac{V_g}{L}$$

$$\hat{i}_L(T_s) = \hat{i}_L(DT_s) + D'T_s\frac{v_c}{L}$$

↑ όχι με χρησιμοποίηση linear ripple approximation

$$\hat{i}_L(T_s) = \hat{i}_L(0) + DT_s\frac{V_g}{L} + D'T_s\frac{v_c}{L}$$

$$\boxed{\hat{i}_L(T_s) = \hat{i}_L(0) + \frac{T_s}{L}\underbrace{(DV_g + D'v_c)}_{\overline{v}_L}}$$

После n периода

$$\hat{i}_L((n+1)T_s) = \hat{i}_L(nT_s) + \frac{T_s}{L}\big(D(nT_s)V_g(nT_s) + D'(nT_s)v(nT_s)\big)$$

Под претпоставком да се V_g и v споро (дискретно) мењају (претпоставке)

Апроксимација извода:

$$\frac{di_L}{dt} \cong \frac{\Delta i_L}{\Delta t} = \frac{i_L((n+1)T_s) - i_L(nT_s)}{T_s}$$

пошто се све споро мења $nT_s \to t$, континуализација

$$\frac{d i_L}{dt} \cong \frac{1}{L}\big(D(t)V_g(t) + D'(t)v_c(t)\big)$$

односно:

$$\boxed{L\frac{di_L}{dt} = DV_g + D'v_c = \bar{v}_L}$$

даље ће се увек писати

Слично за $v_c(t)$:

$$v_c(DT_s) = v(0) - DT_s \frac{v_c}{RC}$$

$$v_c(T_s) = v_c(DT_s) - D' T_s \left(\frac{i_L}{C} + \frac{v_c}{RC} \right)$$

$$v_c(T_s) = v_c(0) - \frac{T_s}{C} \underbrace{\left(\frac{v}{R} + D' i_L \right)}_{= \bar{i}_c}$$

опет прича о n одбирака, апроксимација извода

$$\frac{dv_c}{dt} \cong \frac{\Delta v}{\Delta t} = \frac{v_c((n+1)T_s) - v_c(nT_s)}{T_s}$$

$$\frac{dv_c}{dt} = -\frac{1}{C} \left(\frac{v_c(t)}{R} + D'(t)\, i_L(t) \right)$$

односно

$$\boxed{C \frac{dv_c}{dt} \cong -\frac{v_c}{R} - D' i_L = \bar{i}_c}$$

- Nonlinear large-signal dynamical model

$$\boxed{\begin{aligned} L\frac{di_L}{dt} &= DV_g + D'v_C \\ C\frac{dv_C}{dt} &= -\frac{v_C}{R} - D'i_L \end{aligned}}$$

- Nonlinear: multiplication of time-varying quantities, $D(t)$, $v_c(t)$, $i_L(t)$

- Low-frequency behavior of $i_L(t)$ & $v_c(t)$, ripple filtered out

- Still nonlinear

- Next step: large signal DC and small signal AC models

DC [illegible]

- Quiescent operating point

$$v_g(t) = V_g + \hat{v}_g(t)$$
$$d(t) = D_0 + \hat{d}(t)$$
$$i_L(t) = I_{L0} + \hat{i}_L(t)$$
$$v_c(t) = V_{c0} + \hat{v}_c(t)$$

– DC model (large signal)
transients died out

$$0 = D_0 V_g + D_0' V_c \quad \rightarrow \quad V_c = -\frac{D_0}{D_0'} V_g$$

$$0 = -\frac{V_c}{R} - D_0' I_L \quad \rightarrow \quad I_L = -\frac{V_c}{R D_0'}$$

$\uparrow$

consequence of use of
V sec & A sec balance

– Also called steady-state transfer functions, nonlinear, but who cares

– Perturbation of large-signal dynamical model ($\neq$ large signal DC model)

$$L \frac{d(I_L + \hat{i}_L)}{dt} = (D_0 + \hat{d})(V_g + \hat{v}_g) +$$

$$(D_0' - \hat{d})(V_c + \hat{v}_c)$$

$$L\left(\underbrace{\cancel{\frac{dI_L}{dt}}}_{0} + \frac{d\hat{i}_L}{dt}\right) = \underbrace{(\cancel{D_0 V_g + D_0' V_c})}_{\text{dc terms}}^{0} +$$

$$\underbrace{+ D_0 \hat{v}_g + \hat{d} V_g - \hat{d} V_c + D_0' \hat{v}_c}_{\text{1st order ac terms}} +$$

$$+ \underbrace{\hat{d}\hat{v}_g - \hat{d}\hat{v}_c}_{\text{2nd order ac terms}} \rightarrow \text{neglect}$$

$$L\frac{d\hat{i}_L}{dt} = D_0\hat{v}_g + D_0'\hat{v}_c + \hat{d}(V_g - V_c)$$

- исто за кондензатор

$$C\frac{d(V_c + \hat{v}_c)}{dt} = -\frac{V_c + \hat{v}_c}{R} - (D_0' - \hat{d})(I_L + \hat{i}_L)$$

$$C\frac{d\hat{v}_c}{dt} = \underbrace{\cancel{-\frac{V_c}{R} - D_0' I_L}}_{DC} \rightarrow 0$$

$$\underbrace{-\frac{\hat{v}_c}{R} + \hat{d}I_L - D_0'\hat{i}_L}_{\text{1st order ac}} \quad + \underbrace{\hat{d}\,\hat{i}_L}_{\text{2nd order ac}}$$

→ neglect

$$C\frac{d\hat{v}_c}{dt} = -\frac{\hat{v}_c}{R} - D_0'\hat{i}_L + \hat{d}I_L$$

$$\boxed{\begin{aligned} L\frac{d\hat{i}_L}{dt} &= D_0\hat{v}_g + D_0'\hat{v}_c + \hat{d}(V_g - V_c) \\ C\frac{d\hat{v}_c}{dt} &= -\frac{\hat{v}_c}{R} - D_0'\hat{i}_L + \hat{d}I_L \end{aligned}}$$

small-signal ac model

– Small signal ac models

1 линеаран
2 стационаран
3 ужасно зависи од параметара кола и радне тачке (погрешан често и тип R!)
4 величине са ^ нису фазори
5 фазори се ниже не виде, одређује се са средњим вредностима

– Summary of the models
– large signal dynamic model
– large signal DC model
– small signal AC model

– Summary of Procedure

1° Изразим $i_L(T_s)$ и $v_C(T_s)$ у зависности од $i_L(0)$, $v_C(0)$ и D

2° Апроксимација извода $\frac{di_L}{dt} \approx \frac{i_L(T_s) - i_L(0)}{T_s}$

$\frac{dv_C}{dt} \approx \frac{v_C(T_s) - v_C(0)}{T_s}$; result: nonlinear state equations, nonlinear dynamic model

3° Perturbation and Linearization of nonlinear state equations – small signal ac model

- Систем регулације је по карактеризацији линеаран и стационаран – linear control theory за пројектовање регулатора.

- Пример: преносна функција $\dfrac{\hat{v}_c(s)}{\hat{d}(s)}$

Лапласова трансформација за small signal ac model

$$sL\,\hat{i}_L(s) = D_0'\,\hat{v}_C(s) + (V_g - V_C)\,\hat{d}(s) + D_0\,\cancelto{0}{\hat{v}_g(s)}$$

$$sC\,\hat{v}_C(s) = -D_0'\,\hat{i}_L(s) - \hat{v}_C(s)/R + I_L\,\hat{d}(s)$$

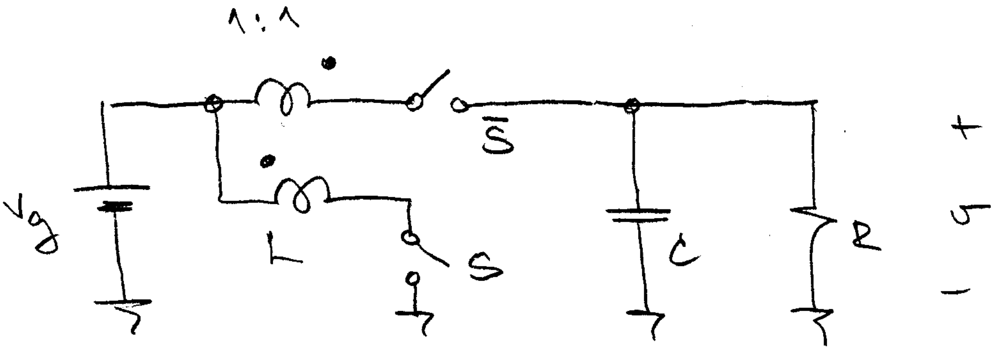
$\hat{v}_g(s) = 0$ – нема варијације улазног напона, линеаран систем, суперпозиција

елиминишемо $\hat{i}_L(s)$, решавамо по $\dfrac{\hat{v}_C(s)}{\hat{d}(s)}$

$$\frac{\hat{v}}{\hat{d}}(s) = -\frac{V_g}{D_0'^2}\,\frac{\left(1 - s\,\dfrac{D_0 L}{D_0'^2 R}\right)}{\left(1 + s\,\dfrac{L}{D_0'^2 R} + s^2\,\dfrac{LC}{D_0'^2}\right)}$$

Домаћи задатак:

Анализа Watkins-Johnson конвертора



1) DC модел

2) AC (small signal) модел

3) реализација S и $\bar{S}$ (кад и који од њих може бити диода)

4) преносне функције

$\frac{\hat{v}}{\hat{d}}(s)$ $\frac{\hat{v}}{\hat{v}_g}(s)$

The State-Space Averaging Method

- претходни метод – perturbation method
- нема тешке разлике, ово је формализам
- генерални метод for ANY converter

- Претпоставке
  1) рипл је "брз", динамика осталих процеса у односу на рипл је спора
  2) конвертор има два стања (може и за више, извођење за два)

- Position 1:

$$\frac{d\underline{x}}{dt} = A_1 \underline{x} + B_1 \underline{u}$$

$$\underline{y} = C_1 \underline{x} + E_1 \underline{u}$$

- Position 2:

$$\frac{d\underline{x}}{dt} = A_2 \underline{x} + B_2 \underline{u}$$

$$\underline{y} = C_2 \underline{x} + E_2 \underline{u}$$

- natural frequencies of converter, regulator, line variations много мање од switching frequency

$$\left.\begin{aligned} \underline{0} &= A\underline{X}_0 + B\underline{U}_0 \\ \underline{Y}_0 &= C\underline{X}_0 + E\underline{U}_0 \end{aligned}\right\} \text{steady state equations}$$

где је

$$A = D_0 A_1 + D_0' A_2$$

$$B = D_0 B_1 + D_0' B_2$$

$$C = D_0 C_1 + D_0' C_2$$

$$E = D_0 E_1 + D_0' E_2$$

$\underline{X}_0$, $\underline{U}_0$, $\underline{Y}_0$, D_0 – у мирној радној тачки

- Ово је нека математичка формулација усредњаваног модела, пошто су две исте једначине

- Small signal ac model

$$\frac{d\hat{x}}{dt} = A\hat{x} + B\hat{u} + \left((A_1 - A_2)\underline{X}_0 + (B_1 - B_2)\underline{U}_0\right)\hat{d}$$

$$\hat{y} = C\hat{x} + E\hat{u} + \left((C_1 - C_2)\underline{X}_0 + (E_1 - E_2)\underline{U}_0\right)\hat{d}$$

- $\hat{x}$, $\hat{y}$, $\hat{u}$, $\hat{d}$ – small signal variations

- увођење раста, усредњавање, елиминација DC чланова, одбацивање 2nd order terms

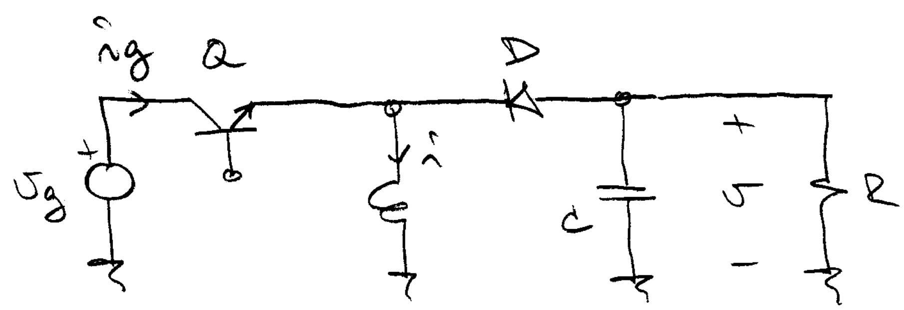
– Изводи се све из међукорака, nonlinear large-signal dynamic model

$$\boxed{\frac{d\underline{x}}{dt} = (DA_1 + D'A_2)\,\underline{x} + (DB_1 + D'B_2)\,\underline{u}}$$

– Вежбање (питања на испиту)

Користећи state-space averaging извести small-signal AC model за boost конвертор.

– Пример Buck-Boost конвертор са губицима, transistor and diode voltage drops V_T & V_D



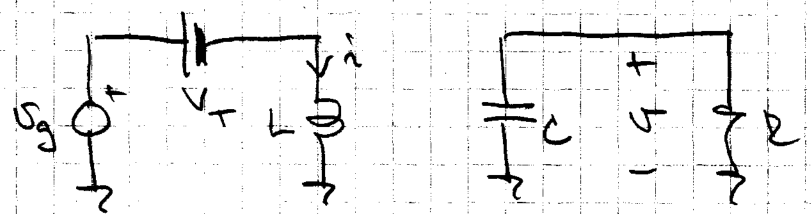
determine both output voltage v and the line current i_g

output vector $\quad \underline{y}(t) = \begin{bmatrix} v(t) \\ i_g(t) \end{bmatrix}$

state vector $\quad \underline{x}(t) = \begin{bmatrix} i(t) \\ v(t) \end{bmatrix}$

input vector $\quad u(t) = \begin{bmatrix} v_g(t) \\ V_T \\ V_D \end{bmatrix}$

Q on



$$L\frac{di}{dt} = v_g - V_T \quad \rightarrow \quad \frac{di}{dt} = \frac{v_g}{L} - \frac{V_T}{L}$$

$$C\frac{dv}{dt} = -\frac{v}{R} \quad \rightarrow \quad \frac{dv}{dt} = -\frac{v}{RC}$$

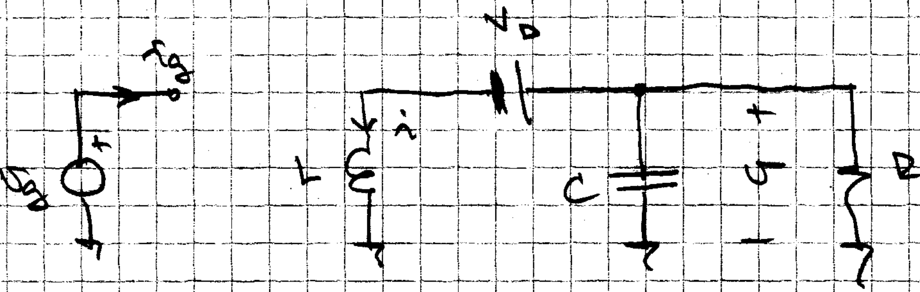
$$i_g = i$$

$$\frac{d\underline{x}}{dt} = \frac{d}{dt}\begin{bmatrix} i \\ v \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}}_{A_1} \underbrace{\begin{bmatrix} i \\ v \end{bmatrix}}_{\underline{x}} +$$

$$\underbrace{\begin{bmatrix} \frac{1}{L} & -\frac{1}{L} & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{B_1} \underbrace{\begin{bmatrix} v_g \\ V_T \\ V_D \end{bmatrix}}_{\underline{u}}$$

$$\underline{y} = \begin{bmatrix} v \\ i_g \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{C_1} \underbrace{\begin{bmatrix} i \\ v \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{E_1} \underbrace{\begin{bmatrix} v_g \\ V_T \\ V_D \end{bmatrix}}_{\underline{u}}$$

Q off



$$L\frac{di}{dt} = v - V_D \qquad \frac{di}{dt} = \frac{v}{L} - \frac{V_D}{L}$$

$$C\frac{dv}{dt} = -\frac{v}{R} - i \qquad \frac{dv}{dt} = -\frac{i}{C} - \frac{v}{RC}$$

$$i_g = 0$$

$$\frac{d}{dt}\begin{bmatrix} i \\ v \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix}}_{A_2} \underbrace{\begin{bmatrix} i \\ v \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} 0 & 0 & -\frac{1}{L} \\ 0 & 0 & 0 \end{bmatrix}}_{B_2} \underbrace{\begin{bmatrix} v_g \\ V_T \\ V_D \end{bmatrix}}_{\underline{u}}$$

$$\begin{bmatrix} v \\ i_g \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{C_2} \underbrace{\begin{bmatrix} i \\ v \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{E_2} \underbrace{\begin{bmatrix} v_g \\ V_T \\ V_D \end{bmatrix}}_{\underline{u}}$$

Сада само применимо готове формуле за усредњене моделе

DC Model
$$\underline{0} = A\underline{X}_0 + B\underline{U}_0$$
$$\underline{Y}_0 = C\underline{X}_0 + E\underline{U}_0$$

AC Model
$$\frac{d\hat{\underline{x}}}{dt} = A\hat{\underline{x}} + B\hat{\underline{u}} + \left((A_1 - A_2)\underline{X}_0 + (B_1 - B_2)\underline{U}_0\right)\hat{d}$$
$$\hat{\underline{y}} = C\hat{\underline{x}} + E\hat{\underline{u}} + \left((C_1 - C_2)\underline{X}_0 + (E_1 - E_2)\underline{U}_0\right)\hat{d}$$

$$A = D_0 A_1 + D_0' A_2$$
$$B = D_0 B_1 + D_0' B_2$$
$$C = D_0 C_1 + D_0' C_2$$
$$E = D_0 E_1 + D_0' E_2$$

Диференциални уравнения:

$$A = \begin{bmatrix} 0 & \frac{D_0'}{L} \\ -\frac{D_0'}{C} & -\frac{1}{RC} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{D_0}{L} & -\frac{D_0}{L} & -\frac{D_0'}{L} \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \\ D_0 & 0 \end{bmatrix}$$

$$A_1 - A_2 = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}$$

$$B_1 - B_2 = \begin{bmatrix} \frac{1}{L} & -\frac{1}{L} & \frac{1}{L} \\ 0 & 0 & 0 \end{bmatrix}$$

$$C_1 - C_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$E_1 - E_2 = \underline{0}$$

$$(A_1 - A_2)\underline{x}_0 + (B_1 - B_2)\underline{u}_0 =$$

$$= \begin{bmatrix} -\frac{V_0}{L} \\ \frac{I_0}{C} \end{bmatrix} + \begin{bmatrix} \frac{V_g - V_T + V_D}{L} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{V_g - V_0 - V_T + V_D}{L} \\ \frac{I_0}{C} \end{bmatrix}$$

$$(C_1 - C_2)\underline{x}_0 + (E_1 - E_2)\underline{u}_0 = \begin{bmatrix} 0 \\ I_0 \end{bmatrix}$$

Модели:

<u>dc model</u>

$$\underline{0} = A\underline{x}_0 + B\underline{u}_0$$

$$\underline{y}_0 = C\underline{x}_0 + E\underline{u}_0$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{D_0'}{L} \\ -\frac{D_0'}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_0 \\ V_0 \end{bmatrix} + \begin{bmatrix} \frac{D_0}{L} & -\frac{D_0}{L} & -\frac{D_0'}{L} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_g \\ V_T \\ V_D \end{bmatrix}$$

$$\underline{y}_0 = \begin{bmatrix} V_0 \\ I_g \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ D_0 & 0 \end{bmatrix} \begin{bmatrix} I_0 \\ V_0 \end{bmatrix} + \underline{0}$$

[illegible]

$$0 = \frac{D_0'}{L} V_0 + \frac{D_0}{L} V_g - \frac{D_0}{L} V_T - \frac{D_0'}{L} V_D$$

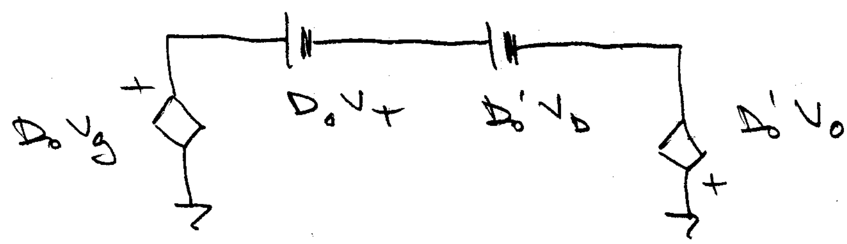
$$0 = D_0' V_0 + D_0 V_g - D_0 V_T - D_0' V_D \qquad (1)$$

$$0 = -\frac{D_0'}{C} I_0 - \frac{V_0}{RC}$$

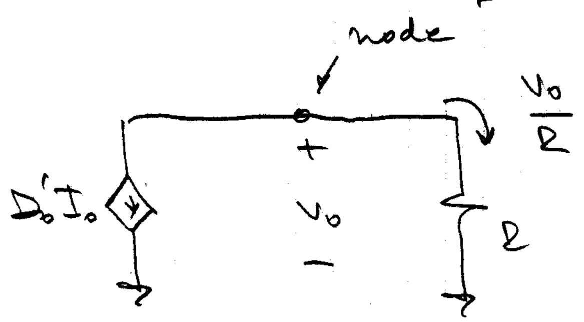
$$0 = D_0' I_0 + \frac{V_0}{R} \qquad (2)$$

$$I_g = D_0 I_0 \qquad (3)$$

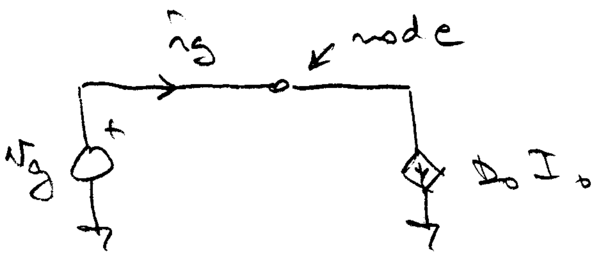
Eq. (1): Loop equation (&3H)



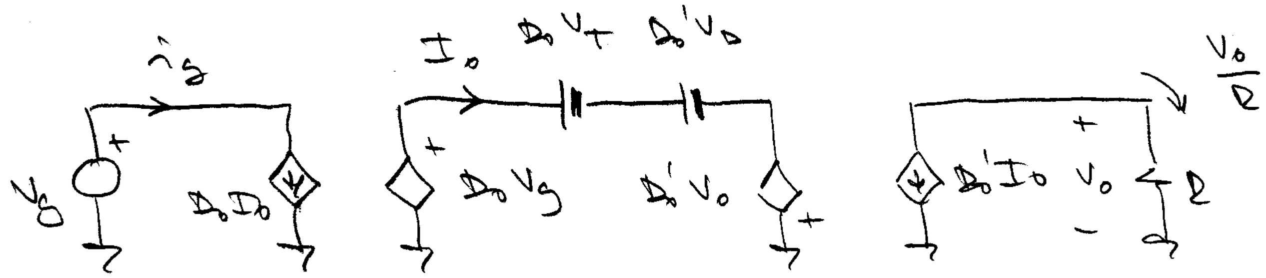
Eq. (2): Node equation (&3C)

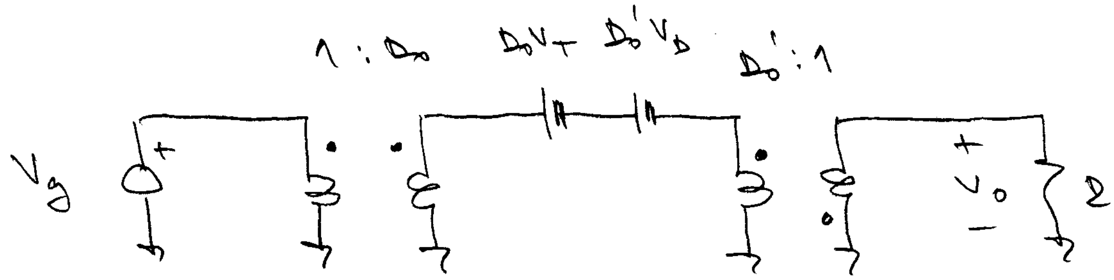


Eq. (3): Node equation (KCL)



Combine all three circuits together





DC transformers

DC model for the Buck-Boost converter

AC Model

$$\frac{d\hat{x}}{dt} = A\hat{x} + B\hat{u} + \left((A_1 - A_2)\underline{X}_0 + (B_1 - B_2)\underline{U}_0\right)\hat{d}$$

$$\frac{d}{dt}\begin{bmatrix}\hat{i}\\ \hat{v}\end{bmatrix} = \begin{bmatrix} 0 & \frac{D_0'}{L} \\ -\frac{D_0'}{C} & -\frac{1}{RC}\end{bmatrix}\begin{bmatrix}\hat{i}\\ \hat{v}\end{bmatrix} +$$

$$+\begin{bmatrix}\frac{D_0}{L} & -\frac{D_0}{L} & -\frac{D_0'}{L} \\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\hat{v}_g \\ \cancel{\hat{v}_T}\; 0 \\ \cancel{\hat{v}_D}\; 0\end{bmatrix} + \begin{bmatrix}\frac{V_g - V_0 - V_T + V_D}{L} \\ \frac{I_0}{C}\end{bmatrix}\hat{d}$$

state relations

$$L\frac{d\hat{i}}{dt} = D_0'\hat{v} + D_0\hat{v}_g + (V_g - V_0 - V_T + V_D)\hat{d} \qquad (4)$$

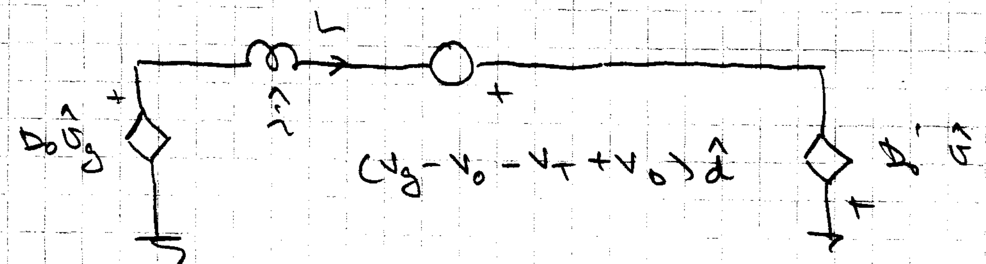
$$C\frac{d\hat{v}}{dt} = -D_0'\hat{i} - \frac{\hat{v}}{R} + I_0\hat{d} \qquad (5)$$

output relations

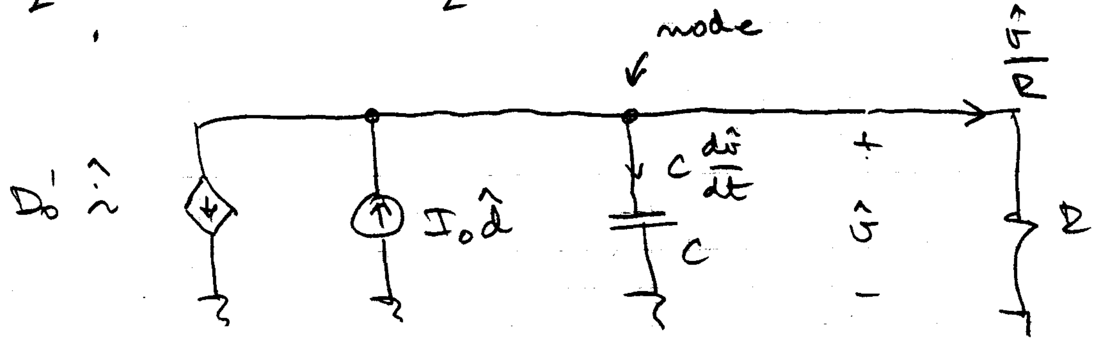
$$\begin{bmatrix}\hat{v}\\ \hat{i}_g\end{bmatrix} = \begin{bmatrix}0 & 1\\ D_0 & 0\end{bmatrix}\begin{bmatrix}\hat{i}\\ \hat{v}\end{bmatrix} + \begin{bmatrix}0\\ I_0\end{bmatrix}\hat{d}$$

$$\hat{i}_g = D_0\hat{i} + I_0\hat{d} \qquad (6)$$

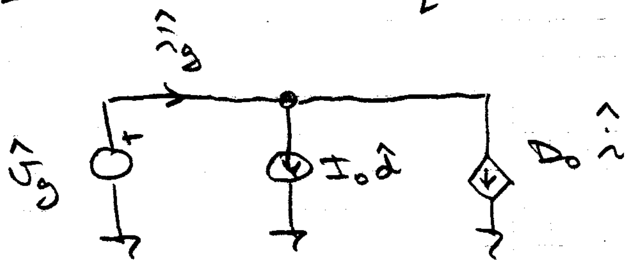
Eq. (4): Loop equation



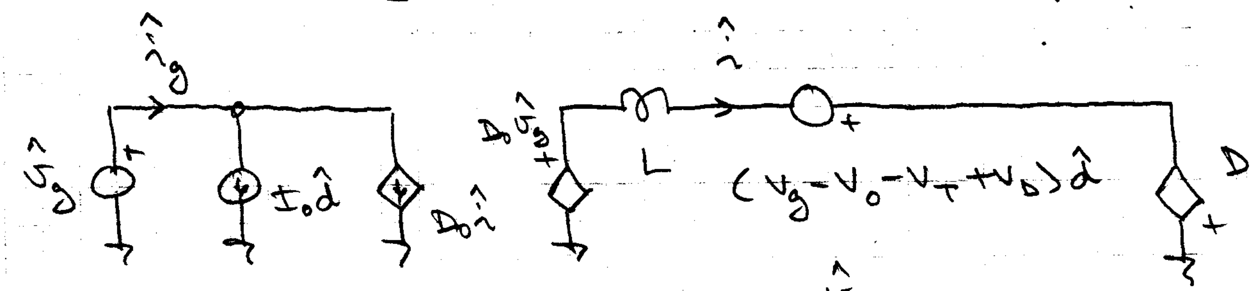
Eq. (5) : Node equation

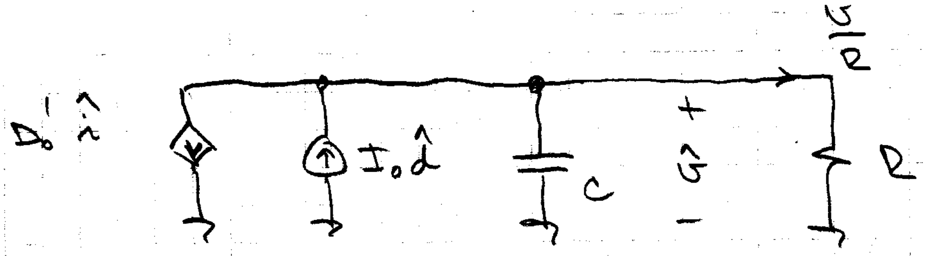


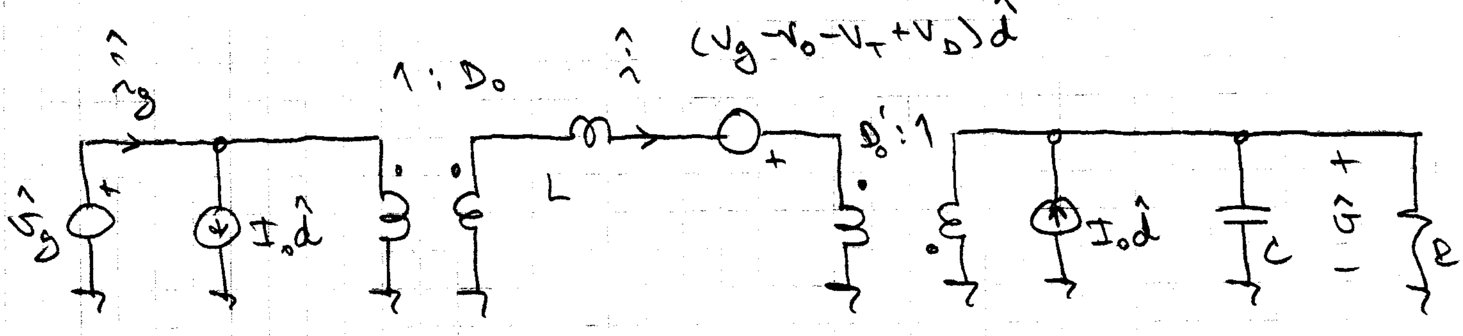
Eq. (6) : Node equation



Combination of all three ac circuit yields





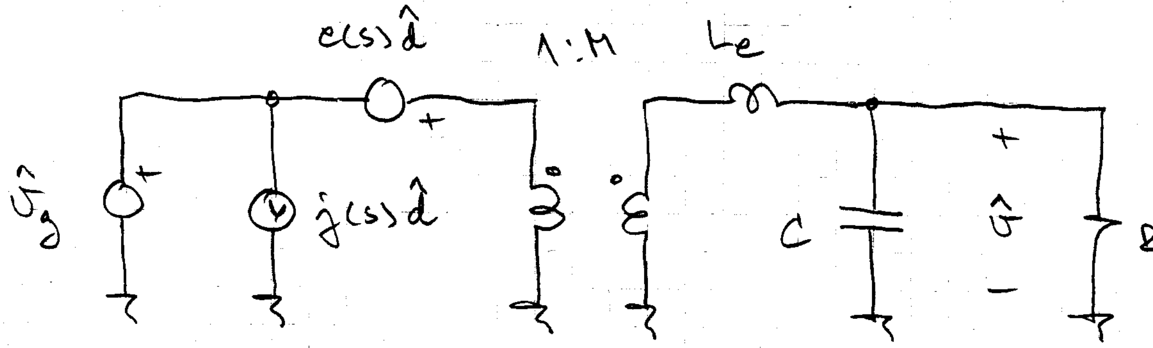


↑ complete ac model

Канонички модел

- општа форма еквивалентног кола за ма који конвертор
- Сви конвертори изводе основне функције:
  1) Трансформација напонских и струјних нивоа, идеално са $\eta = 100\%$
  2) Нискофреквентно филтрирање
  3) Контрола излаза варирањем D

Стога је и канонично еквивалентно коло

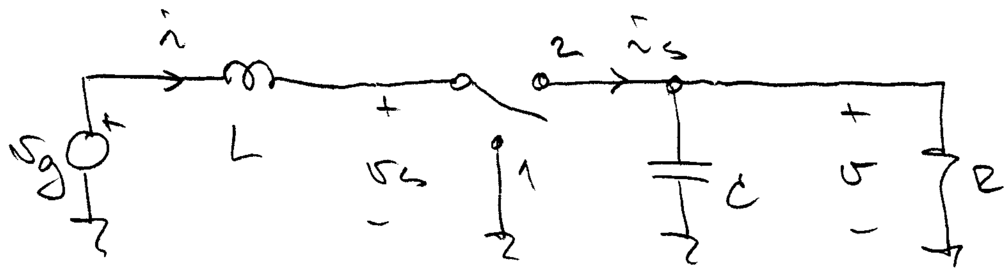


For some common ideal converters

converter	$M(D_0)$	L_e	$e(s)$	$j(s)$
Buck	D_0	L	V_0/D_0^2	V_0/R
Boost	$1/D_0'$	$L/D_0'^2$	$V_0\left(1 - s\frac{L_e}{R}\right)$	$\frac{V_0}{D_0'^2 R}$
Buck-Boost	$-D_0/D_0'$	$L/D_0'^2$	$-\frac{V_0}{D_0^2}\left(1 - s\frac{D_0 L_e}{R}\right)$	$-\frac{V_0}{D_0'^2 R}$

Circuit Averaging

- Исто од state-space averaging
- more physical insight
- користи се за контролирање исправувачи и наизменично-наизменични конвертори
- Пример: Boost converter



Position 1: $v_s = 0$

$i_s = 0$

position 2: $v_s = v$

$i_s = i$

- Averaging:

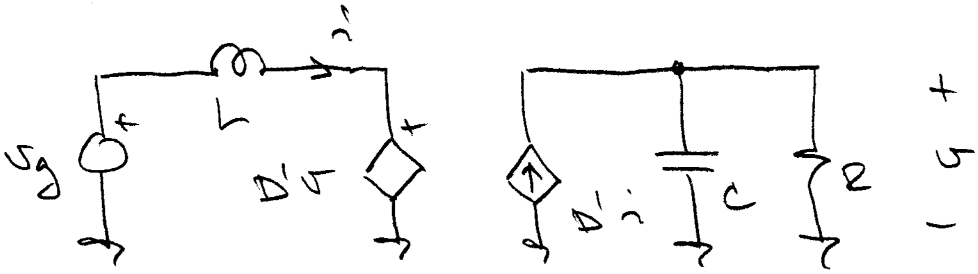
$$\bar{v}_s = \frac{1}{T_s}\left(0 \cdot DT_s + v \cdot D' T_s\right)$$

$$\bar{v}_s = D' v$$

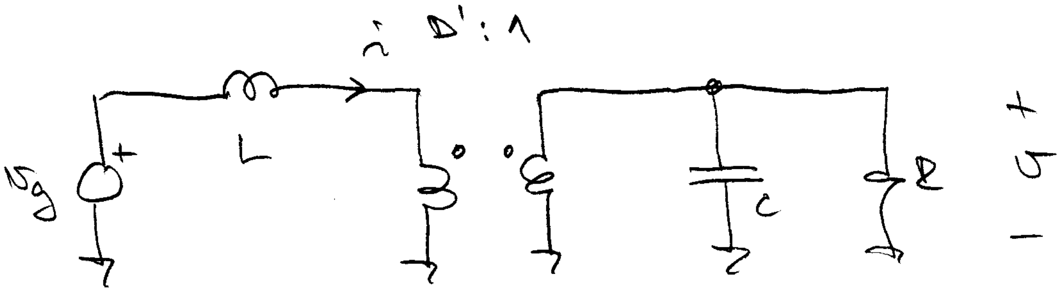
$$\bar{i}_s = \frac{1}{T_s}\left(0 \cdot DT_s + i\, D' T_s\right)$$

$$\bar{i}_s = D' i$$

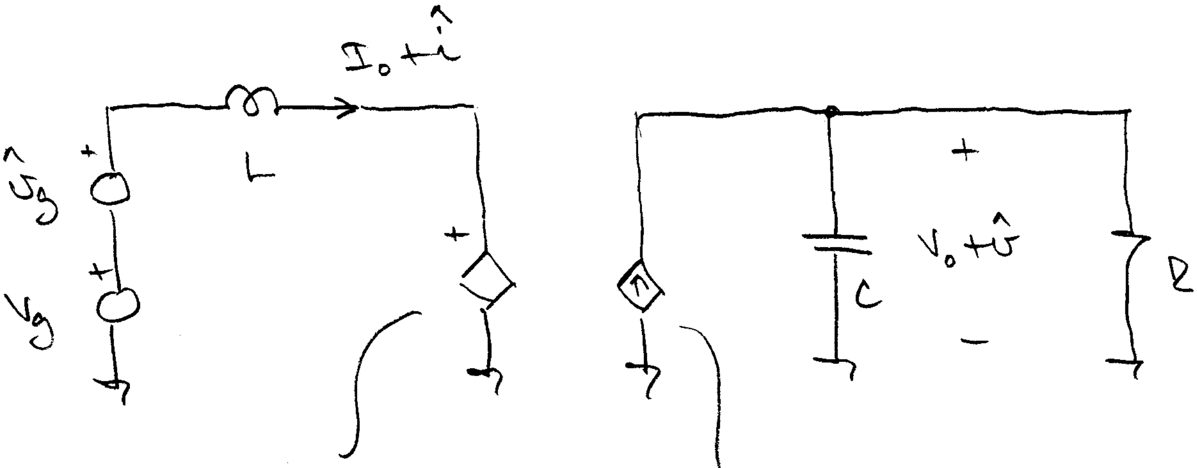
– Averaged circuit



– Примена идеалног трансформатора



– Пертурбација и линеаризација



$(D_o' - \hat{d})(V_o + \hat{v}) =$

$= D_o' V_o +$

$+ D_o' \hat{v} - V_o \hat{d} -$

$- \cancel{\hat{d}\hat{v}} =$

0

$= D_o' (V_o + \hat{v}) -$

$- V_o \hat{d}$

$(D_o' - \hat{d})(I_o + \hat{i}) =$

$= D_o' I_o -$

$- \hat{d} I_o + D_o' \hat{i} -$

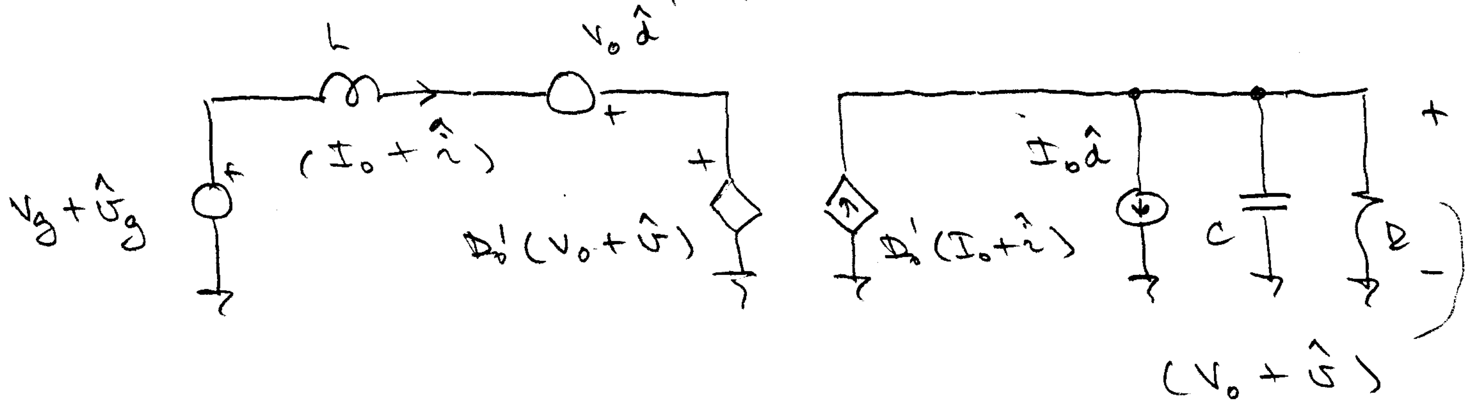
$- \cancel{\hat{d}\hat{i}} =$

0

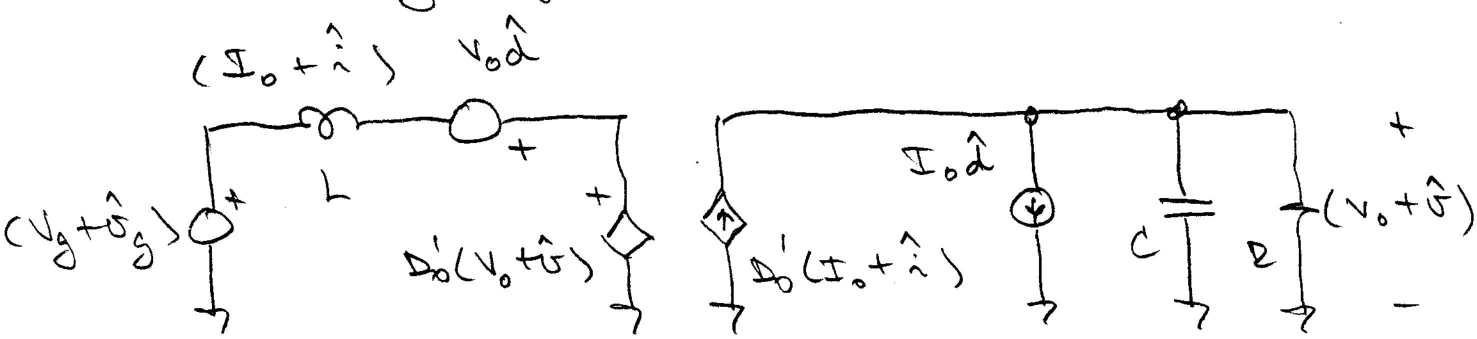
$= D_o' (I_o + \hat{i}) -$

$- \hat{d} I_o$

– физичка интерпретација



Смештамо у трансформатор



(ово је complete averaged & linearized model, canonical form

– Вежба: урадити исто за buck и buck-boost, check за canonical form using circuit averaging

Summary of the circuit averaging procedure

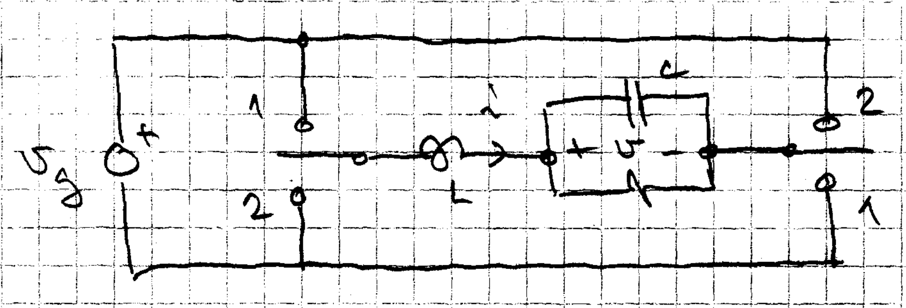
1. Replace switching elements with equivalent current and voltage sources, дефинисани тако да се у колу ништа не догоди.

2. Усреднити the voltages and currents over the switching period

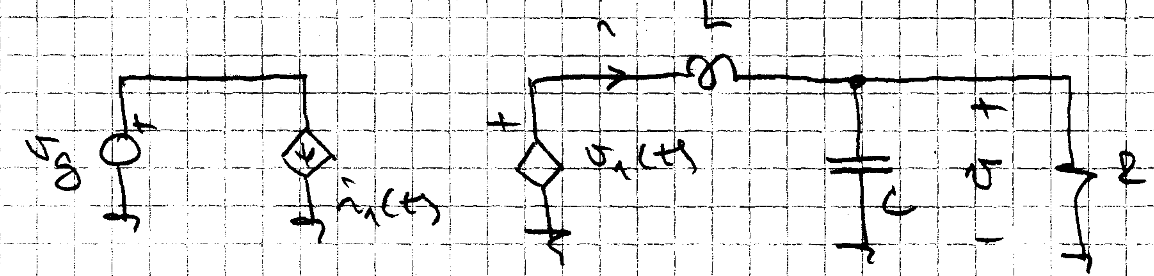
3. Пертурбација и линеаризација, груписање чланова

4. Манипулације да коло добије леп облик

Вежба: Bridge Inverter EXTRA CREDIT



a) Показати да се конвертор може представити као



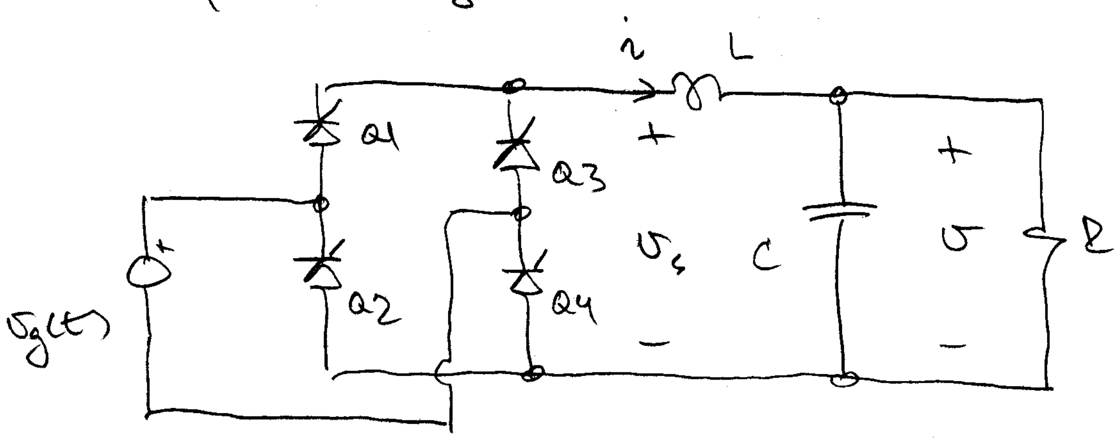
и одредити $i_1(t)$ и $v_1(t)$

в) Извести large-signal circuit-averaged (схематски усредњени) модел овог конвертора

г) Пертурбација и линеаризација, еквивалентни линеаризовани модел

Домаћи задатак:

Small-signal relations in a controlled bridge rectifier



a) small-signal transfer functions

$$\frac{\hat{v}}{\hat{\alpha}}(s), \quad \frac{\hat{v}}{\hat{v}_g}(s)$$

б) steady-state relation

$$V_0 = f(V_{g0}, A)$$

в) equivalent circuit

Page 1c

$i = I + \hat{i}(t)$

$v = V_0 + \hat{v}(t)$

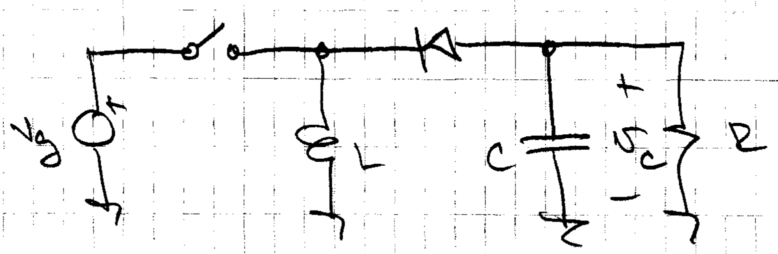
$v_g = (V_{g0} + \hat{v}_g(t)) \sin \omega t$

- Switching frequency $\frac{\omega}{2\pi}$ or $\frac{\omega}{4\pi}$

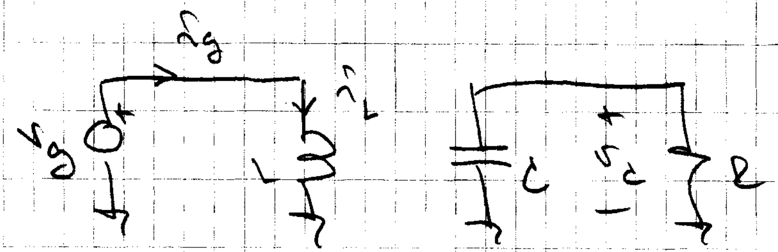
Примјена усредњавања у простору стања на моделовање конвертора у дисконтинуалном моду

- Linear ripple approximation струје калема се не може примјенити, као код DC струје

- Buck - Boost example



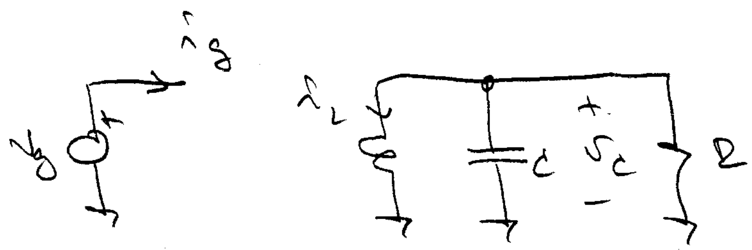
1st Interval, $D_1 T_s$



$$L \frac{di_L}{dt} = V_g \qquad C \frac{dv_C}{dt} = -\frac{v_C}{R}$$

$$i_g = i_L \qquad (1)$$

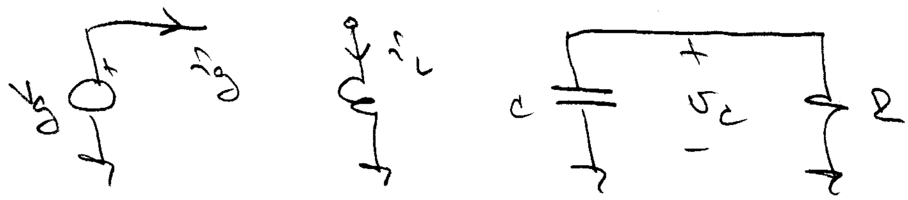
2nd interval $D_2 T_s$



$$L \frac{di_L}{dt} = v_c \qquad C \frac{dv_c}{dt} = -i_L - \frac{v_c}{R} \qquad (2)$$

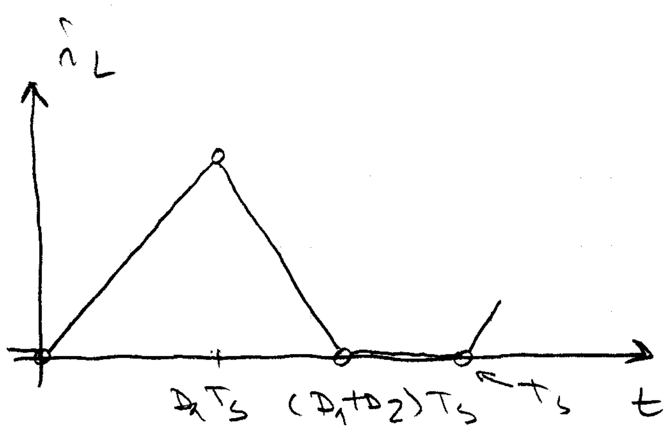
$$i_g = 0$$

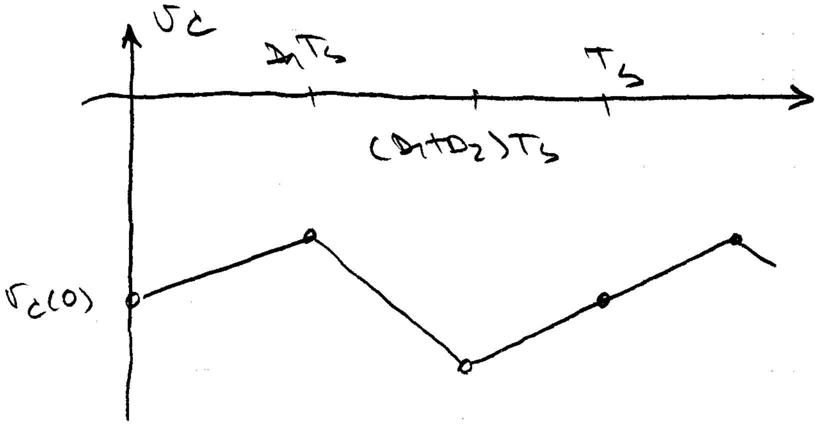
3rd interval $D_3 T_s$



$$L \frac{di_L}{dt} = 0 \;, \; i_L = 0 \qquad C \frac{dv_c}{dt} = -\frac{v_c}{R} \qquad (3)$$

$$i_g = 0$$





1^{st} interval

$$\hat{i}_L(D_1T_s) = \hat{i}_{peak} = \underbrace{\frac{V_g}{L}}_{slope}\underbrace{D_1T_s}_{time} \qquad (\hat{i}_L(0)=0)$$

(4)

$$\hat{i}_g = \hat{i}_L$$

$$v_c(D_1T_s) = v_c(0) - \frac{v_c}{RC} D_1T_s$$

2^{nd} interval

$$\hat{i}_L((D_1+D_2)T_s) = \hat{i}_L(D_1T_s) + \frac{v_c}{L} D_2T_s$$

$$\hat{i}_g = 0$$

(5)

$$v_c((D_1+D_2)T_s) = v_c(D_1T_s) - \underbrace{\left(\frac{\hat{i}_L}{C} + \frac{v_c}{RC}\right)}_{slope} D_2T_s$$

нажалост, $\hat{i}_L$ није константно током D_2T_s, треба узети средњу вредност

$$v_c((D_1+D_2)T_s) \cong v_c(D_1T_s) -$$

$$- \frac{\bar{i}_D|_{D_2T_s} + v_c/R}{C} D_2T_s \cong$$

$$\cong v_c(D_1T_s) - \frac{\frac{1}{2}i_{peak} + \frac{v_c}{R}}{C} D_2T_s \qquad (6)$$

$i_L((D_1+D_2)T_s) = 0$ – 2^{nd} interval ce rely hold

$$0 = i_L(D_1T_s) + \frac{v_c}{L} D_2T_s$$

$$0 = \frac{V_g}{L} D_1T_s + \frac{v_c}{L} D_2T_s$$

$$0 = D_1V_g + D_2v_c \qquad (8)$$

$$\frac{v_c}{V_g} = -\frac{D_1}{D_2} \qquad (9)$$

3^{rd} interval

$$i_L = 0$$
$$i_g = 0 \qquad (10)$$
$$v_c(T_s) = v_c((D_1+D_2)T_s) - \frac{v_c}{RC} D_3T_s$$

Dynamic Equation for output (capacitor) voltage

Комбиновањем Eq. (4), (6) и (10) да би се добило $v_c(T_s)$ у функцији $v_c(0)$ даје

$$v_c(T_s) = v_c(0) - \frac{v_c}{RC} T_s - \frac{\frac{1}{2}\hat{i}_{peak}}{C} D_2 T_s \qquad (11)$$

Апроксимација извода:

$$\frac{dv_c}{dt} \cong \frac{\Delta v_c}{\Delta t} = \frac{v_c(T_s) - v_c(0)}{T_s}$$

From (11)

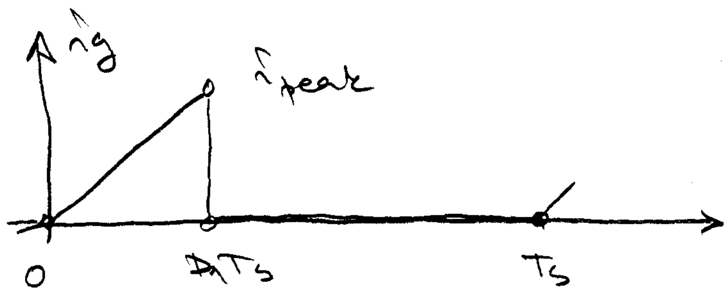
$$\frac{v_c(T_s) - v_c(0)}{T_s} = -\frac{v_c}{RC} - \frac{D_2 \hat{i}_{peak}}{2C} . \qquad (12)$$

$$\hat{i}_{peak} = \frac{V_g}{L} D_1 T_s$$

$$C\frac{dv_c}{dt} = -\frac{v_c}{R} - \frac{D_1 D_2 V_g T_s}{2L} \qquad (13)$$

Equation for the average input current $\bar{i}_g$:

(посебно само у континуалном моделу , input & output properties)



$$\bar{i}_g = \frac{1}{T_s}\int_0^{T_s} i_g(\tau)\,d\tau = \frac{1}{T_s}\left(\frac{1}{2} i_{peak}\right) D_1 T_s =$$

$$= \frac{D_1^2 V_g T_s}{2L} \qquad (14)$$

Имамо :

(15) $\quad D_1 V_g + D_2 v_C = 0$ — из Eq. (8) expression for D_2

(16) $\quad C\dfrac{dv_C}{dt} = -\dfrac{v_C}{R} - \dfrac{D_1 D_2 V_g T_s}{2L}$ — из Eq. (13) charge balance

(17) $\quad \bar{i}_g = \dfrac{D_1^2 V_g T_s}{2L}$ — из Eq. (14) average input current

Ово је нелинеарни систем пошто D_1, D_2, V_g и v_c стоје међусобно измножени у разним комбинацијама

Note: нема expression за $\frac{di_L}{dt}$. $\frac{di_L}{dt} = 0$, пошто $i_L(0) = i_L(T_s) = 0$

Резултат: Преносна карактеристика конвертора има само један пол, зависан од C

Small-Signal AC Equations

- perturb & linearize

$$D_1(t) = D_{10} + \hat{d}_1(t)$$
$$D_2(t) = D_{20} + \hat{d}_2(t)$$
$$v_g(t) = V_{g0} + \hat{v}_g(t)$$
$$v_c(t) = V_{c0} + \hat{v}_c(t)$$
$$\bar{i}_g(t) = I_{g0} + \hat{i}_g(t)$$

Eq. (15)

$$(D_{10} + \hat{d}_1)(V_{g0} + \hat{v}_g) + (D_{20} + \hat{d}_2)(V_{c0} + \hat{v}_c) = 0$$

$$0 = \underbrace{(D_{10} V_{g0} + D_{20} V_{c0})}_{\text{DC terms}} +$$

$$+ \underbrace{D_{10}\hat{v}_g + V_{g0}\hat{d}_1 + D_{20}\hat{v}_c + V_{c0}\hat{d}_2}_{\text{linear AC}} +$$

$$+ \underbrace{\hat{d}_1\hat{v}_g + \hat{d}_2\hat{v}_c}_{\text{nonlinear}}$$

Eq. (16) becomes

$$C\frac{d}{dt}(\cancel{V_{c0}} + \hat{v}_c) = -\frac{V_{c0}+\hat{v}_c}{R} - \frac{(D_{10}+\hat{d}_1)(D_{20}+\hat{d}_2)(V_{g0}+\hat{v}_g)T_s}{2L}$$

$$C\frac{d\hat{v}_c}{dt} = \underbrace{-\frac{V_{c0}}{R} - \frac{D_{10}D_{20}V_{g0}T_s}{2L}}_{\text{dc}} -$$

$$\underbrace{-\frac{D_{10}D_{20}\hat{v}_gT_s}{2L} - \frac{D_{10}\hat{d}_2V_{g0}T_s}{2L} - \frac{\hat{v}_c}{R}}_{\text{linear ac}} -$$

$$\underbrace{-\frac{\hat{d}_1D_{20}V_{g0}T_s}{2L}}_{\text{linear ac}} - \text{nonlinear terms}$$

DC terms

$$0 = D_{10} V_{g0} + D_{20} V_{c0}$$ expression for D_{20}

$$0 = -\frac{V_{c0}}{R} - \frac{D_{10} D_{20} V_{g0} T_s}{2L}$$ charge balance

$$I_{g0} = \frac{D_{10}^2 V_{g0} T_s}{2L}$$ input current

3 equations, 3 unknowns (D_{20}, V_{c0}, I_{g0})

Solution:

$$D_{20} = \sqrt{K} \qquad \text{where} \qquad K = \frac{2L}{R T_s}$$

$$V_{c0} = -\frac{V_{g0} D_{10}}{\sqrt{K}}$$

$$I_{g0} = \frac{D_{10}^2}{K} \frac{V_{g0}}{R}$$

Small-Signal AC Terms

$$0 = D_{10}\hat{v}_g + V_{g0}\hat{d}_1 + D_{20}\hat{v}_c + V_{c0}\hat{d}_2 \qquad \text{(exp. for } \hat{d}_2\text{)}$$

$$C\frac{d\hat{v}_c}{dt} = -\frac{\hat{v}_c}{R} - \frac{T_s}{2L}\left(D_{10}D_{20}\hat{v}_g + D_{10}V_{g0}\hat{d}_2 + D_{20}V_{g0}\hat{d}_1\right)$$

(capacitor charge)

$$\hat{i}_g = \frac{D_{10}^2 T_s}{2L}\hat{v}_g + \frac{D_{10}V_{g0}T_s}{L}\hat{d}_1 =$$

$$= \frac{D_{10}^2}{RK}\hat{v}_g + \frac{2D_{10}V_{g0}}{RK}\hat{d}_1 \qquad \text{(input current)}$$

$$\hat{d}_2 = -\frac{D_{10}\hat{v}_g + V_{g0}\hat{d}_1 + D_{20}\hat{v}_c}{V_{c0}}$$

→ yields: elimination

$\hat{d}_2$ → [illegible] $\hat{j}$-tie

$$C\frac{d\hat{v}_c}{dt} = -\frac{\hat{v}_c}{R} - \frac{T_s}{2L}\left(\hat{v}_g\left(D_{10}D_{20} - D_{10}^2\frac{V_{g0}}{V_{c0}}\right) + \right.$$

$$\left. + \hat{d}_1\left(D_{20}V_{g0} - D_{10}\frac{V_{g0}^2}{V_{c0}}\right) - \hat{v}_c D_{10}D_{20}\frac{V_{g0}}{V_{c0}}\right)$$

$$C\frac{d\hat{v}_c}{dt} = -\hat{v}_c\left(\frac{1}{R} + \frac{1}{R}\right) + \hat{v}_g\frac{2D_{10}}{R\sqrt{K}} + \hat{d}_1\frac{2V_{g0}}{R\sqrt{K}}$$

↑

[illegible]

dynamics

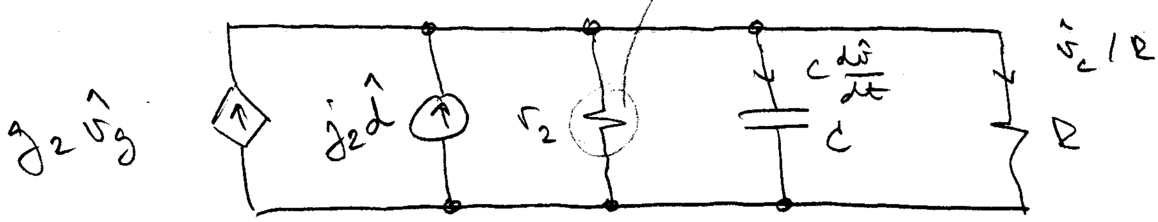
Equivalent circuit model

Output relation

$$C\frac{d\hat{v}_c}{dt} = -\hat{v}_c\left(\frac{1}{R}+\frac{1}{R}\right) + \hat{v}_g\,\frac{2D_{10}}{R\sqrt{K}} + \hat{d}_1\,\frac{2V_{g0}}{R\sqrt{K}}$$

a node equation

r_2 — ac resistance

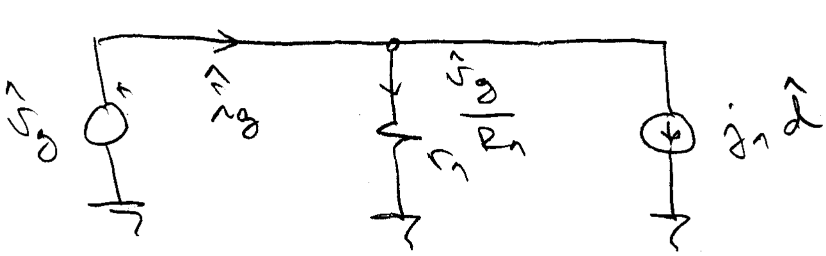


$$g_2 = \frac{2D_{10}}{R\sqrt{K}}, \quad j_2 = \frac{2V_{g0}}{R\sqrt{K}}, \quad r_2 = R$$

[illegible] ac resistance

Input relation

$$\hat{i}_g = \frac{D_{10}^2}{RK}\hat{v}_g + \frac{2D_{10}V_{g0}}{RK}\hat{d}_1$$



$$r_1 = R\frac{K}{D_{10}^2}$$

$$j_1 = \frac{V_{g0}}{R}\,\frac{2D_{10}}{K}$$

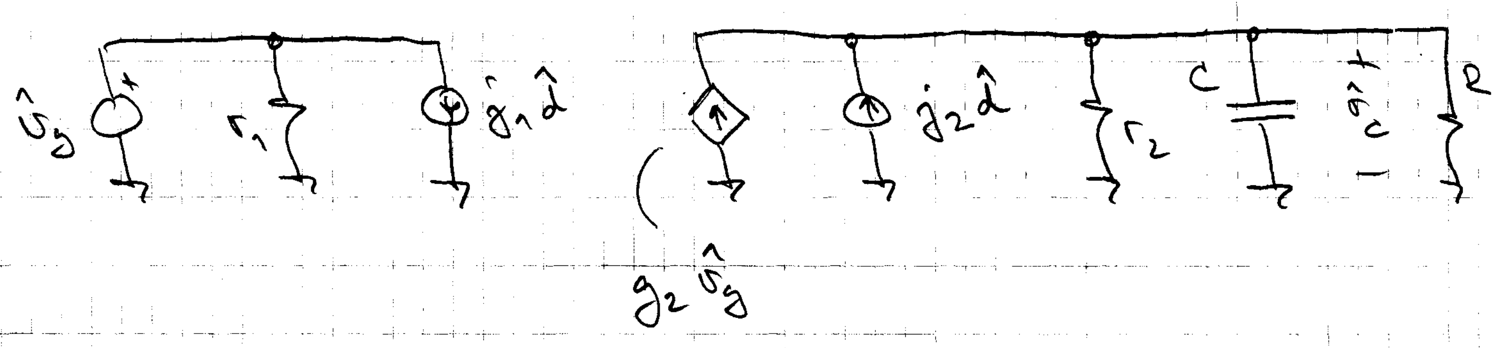
ohmic ac resistor

r_1 & r_2 are effective ac resistances only;

they do not enter into the dc model, nor they

represent power loss

Complete ac small-signal model for
buck-boost in discontinuous mode



r_1 & r_2 are ac resistors

Summary of the Analysis Technique

1 Must determine $v(T_s)$ in terms of $v(0)$, then approximate $\frac{dv}{dt} = \frac{v(T_s) - v(0)}{T_s}$

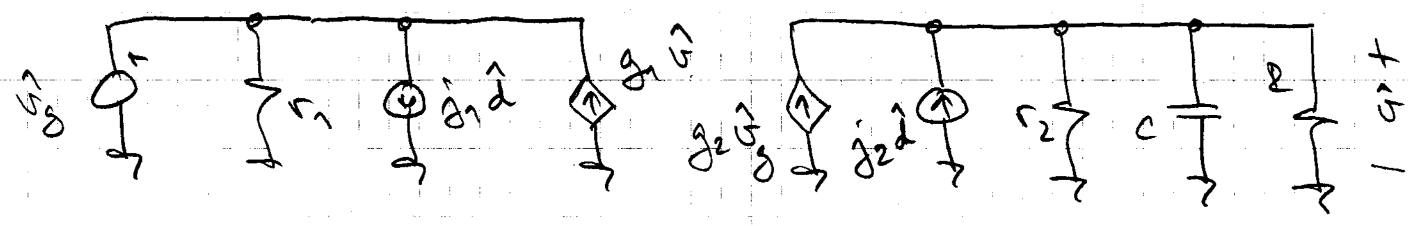
2 Cannot approximate $i(t)$ as nearly constant, since inductor current ripple is not small. Instead, use average value of i.

3 An additional equation is needed to determine the length of the second interval $D_2 T_s$: The second interval ends when $i((D_1 + D_2) T_s) = 0$.

Result: Nonlinear Diff. Eq. which describe $\frac{dv}{dt}$. No equation for $\frac{di}{dt}$, since $i(T_s) = i(0) = 0 \Rightarrow \frac{di}{dt} = 0$, and the inductor does not contribute a pole to the converter dynamics.

- One can construct small-signal ac model by perturbation and linearization, as usual

Canonical Circuit Model – DCM

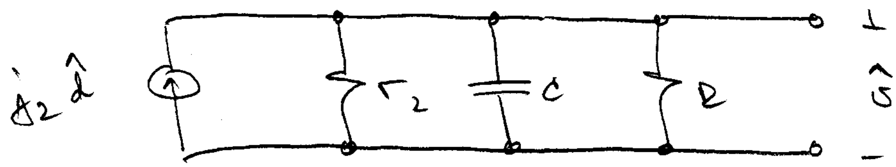


converter	r_1	j_1	g_1	g_2	j_2	r_2
buck	$\frac{1-M}{M^2}R$	$\frac{2V_o}{R}\sqrt{\frac{1-M^2}{K}}$	$\frac{M^2}{1-M}\ \frac{1}{R}$	$\frac{M(2-M)}{(1-M)R}$	$\frac{2V_o}{RM}\sqrt{\frac{1-M}{K}}$	$(1-M)R$
boost	$\frac{M-1}{M^2}R$	$\frac{2V_o}{R}\sqrt{\frac{M}{K(M-1)}}$	$\frac{M}{M-1}\ \frac{1}{R}$	$\frac{M(2M-1)}{(M-1)R}$	$\frac{2V_o}{R\sqrt{KM(M-1)}}$	$\frac{M-1}{M}R$
buck-boost	$\frac{R}{M^2}$	$\frac{2\lvert V_o\rvert}{R\sqrt{K}}$	0	$\frac{2M}{R}$	$\frac{2\lvert V_o\rvert}{R\sqrt{K}M}$	R

$$M = \frac{V_o}{V_g} = \begin{cases} \dfrac{2}{1+\sqrt{1+4K/D_1^2}} & \text{buck} \\[2ex] \dfrac{1+\sqrt{1+4D_1^2/K}}{2} & \text{boost} \\[2ex] -\dfrac{D_1}{\sqrt{K}} & \text{buck-boost} \end{cases}$$

Small-signal transfer functions are easily found from the model:

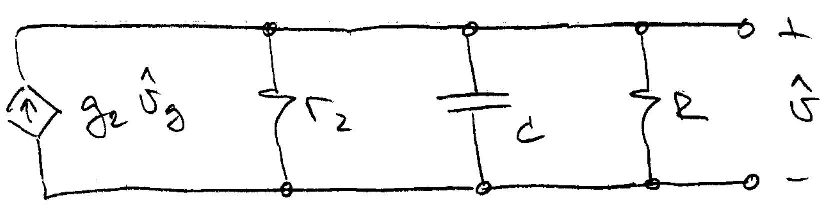
a) $\dfrac{\hat v}{\hat d}$: $\hat v_g = 0$



$$\hat v = j_2 \hat d \left(r_2 \parallel \frac{1}{sC} \parallel R \right)$$

$$\frac{\hat v}{\hat d} = G_{do} \frac{1}{1 + \frac{s}{\omega_p}}$$ — [illegible]

b) $\dfrac{\hat v}{\hat v_g}$: set $\hat d = 0$



$$\hat v = g_2 \hat v_g \left(r_2 \parallel \frac{1}{sC} \parallel R \right)$$

$$\frac{\hat v}{\hat v_g} = \frac{G_{go}}{1 + \frac{s}{\omega_p}}$$

b) Z_{out} : $\hat{v}_g = 0$, $\hat{d} = 0$

$$Z_{out} = r_2 \parallel \frac{1}{sC} \parallel R$$