Formule koje je dozvoljeno koristiti na ispitu

Furijeova transformacija: $X\left(j\omega\right)=\sum_{n=-\infty}^{+\infty}x[n]e^{-j\omega n}$

Inverzna Furijeova transformacija: $x[n]=\frac{1}{2\pi}\int_{2\pi}X\left(j\omega\right)e^{j\omega n}d\omega$

Z-transformacija: $X\left(z\right)=\sum_{n=-\infty}^{+\infty}x[n]z^{-n}$

Inverzna Z-transformacija: $x[n]=\frac{1}{2\pi j}\oint_{C}X\left(z\right)z^{n-1}dz$

Laplasova transformacija: $X\left(s\right)=\int_{0}^{\infty}x(t)e^{-st}dt$

Inverzna Laplasova transformacija: $x\left(t\right)=\frac{1}{2\pi j}\int_{\sigma_0-j\infty}^{\sigma_0+j\infty}e^{st}X\left(s\right)ds$

Diskretna Furijeova transformacija: $X\left[k\right]=\sum_{n=0}^{N-1}x[n]e^{-j2\pi kn/N}$

Inverzna diskretna Furijeova transformacija: $x[n]=\frac{1}{N}\sum_{n=0}^{N-1}X\left[k\right]e^{j2\pi kn/N}$

Trougaona prozorska funkcija: $w_T[n]=\begin{cases}2n/N, & n=0,1,2,\ldots,N/2\\ w_T[N-n], & n=(N/2)+1,(N/2)+2,\ldots,N-1\end{cases}$

Hanova prozorska funkcija: $w_H[n]=\sin^2\left(\frac{n\pi}{N}\right)=\frac{1}{2}\left[1-\cos\left(\frac{2n\pi}{N}\right)\right]$

Hemingova prozorska funkcija: $w_H[n]=\alpha_H-\left(1-\alpha_H\right)\cos\left(\frac{2n\pi}{N}\right)$

Blekmenova prozorska funkcija: $w_B[n]=\sum_{m=0}^{M}\left(-1\right)^m a_m\cos\left(\frac{2nm\pi}{N}\right),\sum_{m=0}^{M}a_m=1$

Veze između slabljenja u linearnoj razmeri i razmeri u decibelima:

$\alpha_p=20\log\frac{1+\delta_p}{1-\delta_p}$ ili $\alpha_p=20\log\frac{1}{1-\delta_p}$, $\alpha_a=-20\log\delta_a$

Kvadrat modula funkcije prenosa analognog Batervortovog NF filtra: $H_B\left(s\right)H_B\left(-s\right)=\frac{1}{1+\varepsilon^2\left(s/j\omega_p\right)^{2N}}$

Transformacije učestanosti:

NF→PO: $H_{PO}\left(\hat{s}\right)=H_{NF}\left(s\right)\Big|_{s=\frac{\hat{s}^2+\omega_0^2}{B\hat{s}}}$

NF→NO: $H_{NO}\left(\hat{s}\right)=H_{NF}\left(s\right)\Big|_{s=\frac{B\hat{s}}{\hat{s}^2+\omega_0^2}}$

Bilinearna transformacija, smena: $s=\frac{2}{T}\frac{z-1}{z+1}$

Furijeova transformacija

Vremenski domen x(t)	**Frekvetni domen X(ω)**
$x(t)$	$X(\omega)=\int_{-\infty}^{\infty} x(t)\mathrm{e}^{-j\omega t}\,dt=\mathfrak{F}\{x(t)\}$
$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\mathrm{e}^{j\omega t}\,d\omega=\mathfrak{F}^{-1}\{X(\omega)\}$	$X(\omega)$
$\delta(t)$	1
1	$2\pi\delta(\omega)$
$\cos(\omega_0 t+\theta)$	$\pi\left[e^{j\theta}\delta(\omega-\omega_0)+e^{-j\theta}\delta(\omega+\omega_0)\right]$
$\sin(\omega_0 t+\theta)$	$-j\pi\left[e^{j\theta}\delta(\omega-\omega_0)-e^{-j\theta}\delta(\omega+\omega_0)\right]$
$\exp(j\omega_0 t)$	$2\pi\delta(\omega-\omega_0)$
$u(t)\equiv\begin{cases}1 & t\ge 0\\ 0 & t<0\end{cases}$	$\pi\delta(\omega)+\frac{1}{j\omega}$
$\operatorname{sgn}(t)\equiv\begin{cases}1 & t\ge 0\\ -1 & t<0\end{cases}$	$\frac{2}{j\omega}$
$1/t$	$-j\pi\operatorname{sgn}(\omega)$
$\sum_{n=-\infty}^{\infty}\delta(t-nT_s)$	$\frac{2\pi}{T_s}\sum_{k=-\infty}^{\infty}\delta\left(\omega-k\frac{2\pi}{T_s}\right)$
$\sum_{k=-\infty}^{\infty}a_k e^{jk\omega_0 t}$, gde je $a_k=\frac{1}{T_0}\int_{T_0}x(t)e^{-jk\omega_0 t}dt$	$2\pi\sum_{k=-\infty}^{\infty}a_k\delta(\omega-k\omega_0)$

Tablica parova Furijeove transformacije diskretnih signala

Signal	DTFT
$1, \quad -\infty < n < \infty$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$
$\mathrm{sgn}[n] = \begin{cases} -1, & \ldots, -3, -2, -1 \\ 1, & 0, 1, 2, \cdots \end{cases}$	$\dfrac{2}{1 - e^{-j\Omega}}$
$u[n]$	$\dfrac{1}{1 - e^{-j\Omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$
$\delta[n]$	$1, \quad -\infty < \Omega < \infty$
$\delta[n-q], \quad q = \pm 1, \pm 2, \pm 3, \ldots$	$e^{-jq\Omega}, \quad q = \pm 1, \pm 2, \pm 3, \ldots$
$a^n u[n], \quad \lvert a \rvert < 1$	$\dfrac{1}{1 - a e^{-j\Omega}}, \quad \lvert a \rvert < 1$
$e^{j\Omega_o n}, \quad \Omega_o$ real	$2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - \Omega_o - 2\pi k), \quad \Omega_o$ real
$p_q[n] = \begin{cases} 1, & n = -q, -q+1, \ldots \\ & , -1, 0, 1, \ldots q \\ 0, & otherwise \end{cases}$	$\dfrac{\sin\left[\left(q + \frac{1}{2}\right)\Omega\right]}{\sin(\Omega / 2)}$
$\frac{B}{\pi} \mathrm{sinc}\left[\frac{B}{\pi} n\right]$	$\sum_{k=-\infty}^{\infty} p_{2B}(\Omega + 2\pi k)$
$\cos(\Omega_o n)$	$\pi \sum_{k=-\infty}^{\infty} \left[\delta(\Omega + \Omega_o - 2\pi k) + \delta(\Omega - \Omega_o - 2\pi k)\right]$
$\cos(\Omega_o n + \theta)$	$\pi \sum_{k=-\infty}^{\infty} \left[e^{-j\theta} \delta(\Omega + \Omega_o - 2\pi k) + e^{j\theta} \delta(\Omega - \Omega_o - 2\pi k)\right]$
$\sin(\Omega_o n)$	$j\pi \sum_{k=-\infty}^{\infty} \left[\delta(\Omega + \Omega_o - 2\pi k) - \delta(\Omega - \Omega_o - 2\pi k)\right]$
$\sin(\Omega_o n + \theta)$	$j\pi \sum_{k=-\infty}^{\infty} \left[e^{-j\theta} \delta(\Omega + \Omega_o - 2\pi k) - e^{j\theta} \delta(\Omega - \Omega_o - 2\pi k)\right]$

Osobine Furijeove transformacije diskretnih signala

Property Name	**Property**	
Linearity	$ax[n]+bv[n]$	$aX(\Omega)+bV(\Omega)$
Time Shift	$x[n-q]$, q any integer	$e^{-jq\Omega}X(\Omega)$, q any integer
Time Scaling	$x(at)$, $a\neq 0$	$\frac{1}{a}X(\Omega/a)$, $a\neq 0$
Time Reversal	$x[-n]$	$X(-\Omega)$ $\overline{X(\Omega)}$ if $x[n]$ is real
Multiply by n	$nx[n]$	$j\frac{d}{d\Omega}X(\Omega)$
Multiply by Complex Exponential	$e^{j\Omega_o n}x[n]$, Ω_o real	$X(\Omega-\Omega_o)$, Ω_o real
Multiply by Sine	$\sin(\Omega_o n)x[n]$	$\frac{j}{2}\left[X(\Omega+\Omega_o)-X(\Omega-\Omega_o)\right]$
Multiply by Cosine	$\cos(\Omega_o n)x[n]$	$\frac{1}{2}\left[X(\Omega+\Omega_o)+X(\Omega-\Omega_o)\right]$
Summation	$\sum_{i=-\infty}^{n}x[i]$	$\frac{1}{1-e^{-j\Omega}}X(\Omega)+\pi\sum_{k=-\infty}^{\infty}X(0)\delta(\Omega-2\pi k)$
Parseval's Theorem (General)	$\sum_{n=-\infty}^{\infty}x[n]\overline{v[n]}=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(\Omega)\overline{V(\Omega)}d\Omega$	
Parseval's Theorem (Energy)	$\sum_{n=-\infty}^{\infty}x^2[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}\lvert X(\Omega)\rvert^2 d\Omega$ if $x(t)$ is real $\sum_{n=-\infty}^{\infty}\lvert x[n]\rvert^2=\frac{1}{2\pi}\int_{-\pi}^{\pi}\lvert X(\Omega)\rvert^2 d\Omega$	
Using CTFT Table to find Inverse of a DTFT $X(\Omega)$: $x[n]$ = ??	Form $\Gamma(\omega)=X(\omega)p_{2\pi}(\omega)$ and look up $\gamma(t)\leftrightarrow\Gamma(\omega)$ Then get $x[n]=\gamma(t)\vert_{t=n}$	

Tabela Laplasovih i Z transformacija

	$X(s)$	$x(t)$	$x(kT)$ or $x[k]$	$X(z)$
1.	–	–	Kronecker delta $\delta_0(k)$ 1 $k = 0$ 0 $k \neq 0$	1
2.	–	–	$\delta_0(n-k)$ 1 $n = k$ 0 $n \neq k$	z^{-k}
3.	$\frac{1}{s}$	$1(t)$	$1[k]$	$\frac{1}{1-z^{-1}}$
4.	$\frac{1}{s+a}$	e^{-at}	e^{-akT}	$\frac{1}{1-e^{-aT}z^{-1}}$
5.	$\frac{1}{s^2}$	t	kT	$\frac{Tz^{-1}}{\left(1-z^{-1}\right)^2}$
6.	$\frac{2}{s^3}$	t^2	$(kT)^2$	$\frac{T^2z^{-1}\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^3}$
7.	$\frac{6}{s^4}$	t^3	$(kT)^3$	$\frac{T^3z^{-1}\left(1+4z^{-1}+z^{-2}\right)}{\left(1-z^{-1}\right)^4}$
8.	$\frac{a}{s(s+a)}$	$1 - e^{-at}$	$1 - e^{-akT}$	$\frac{\left(1-e^{-aT}\right)z^{-1}}{\left(1-z^{-1}\right)\left(1-e^{-aT}z^{-1}\right)}$
9.	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at} - e^{-bt}$	$e^{-akT} - e^{-bkT}$	$\frac{\left(e^{-aT}-e^{-bT}\right)z^{-1}}{\left(1-e^{-aT}z^{-1}\right)\left(1-e^{-bT}z^{-1}\right)}$
10.	$\frac{1}{(s+a)^2}$	te^{-at}	kTe^{-akT}	$\frac{Te^{-aT}z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^2}$
11.	$\frac{s}{(s+a)^2}$	$(1 - at)e^{-at}$	$(1 - akT)e^{-akT}$	$\frac{1-\left(1+aT\right)e^{-aT}z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^2}$
12.	$\frac{2}{(s+a)^3}$	t^2e^{-at}	$(kT)^2e^{-akT}$	$\frac{T^2e^{-aT}\left(1+e^{-aT}z^{-1}\right)z^{-1}}{\left(1-e^{-aT}z^{-1}\right)^3}$
13.	$\frac{a^2}{s^2(s+a)}$	$at - 1 + e^{-at}$	$akT - 1 + e^{-akT}$	$\frac{\left[\left(aT-1+e^{-aT}\right)+\left(1-e^{-aT}-aTe^{-aT}\right)z^{-1}\right]z^{-1}}{\left(1-z^{-1}\right)^2\left(1-e^{-aT}z^{-1}\right)}$
14.	$\frac{\omega}{s^2+\omega^2}$	$\sin \omega t$	$\sin \omega kT$	$\frac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
15.	$\frac{s}{s^2+\omega^2}$	$\cos \omega t$	$\cos \omega kT$	$\frac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T+z^{-2}}$
16.	$\frac{\omega}{(s+a)^2+\omega^2}$	$e^{-at}\sin \omega t$	$e^{-akT}\sin \omega kT$	$\frac{e^{-aT}z^{-1}\sin\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
17.	$\frac{s+a}{(s+a)^2+\omega^2}$	$e^{-at}\cos \omega t$	$e^{-akT}\cos \omega kT$	$\frac{1-e^{-aT}z^{-1}\cos\omega T}{1-2e^{-aT}z^{-1}\cos\omega T+e^{-2aT}z^{-2}}$
18.	–	–	a^k	$\frac{1}{1-az^{-1}}$
19.	–	–	a^{k-1} $k = 1, 2, 3, \ldots$	$\frac{z^{-1}}{1-az^{-1}}$
20.	–	–	ka^{k-1}	$\frac{z^{-1}}{\left(1-az^{-1}\right)^2}$
21.	–	–	k^2a^{k-1}	$\frac{z^{-1}\left(1+az^{-1}\right)}{\left(1-az^{-1}\right)^3}$
22.	–	–	k^3a^{k-1}	$\frac{z^{-1}\left(1+4az^{-1}+a^2z^{-2}\right)}{\left(1-az^{-1}\right)^4}$
23.	–	–	k^4a^{k-1}	$\frac{z^{-1}\left(1+11az^{-1}+11a^2z^{-2}+a^3z^{-3}\right)}{\left(1-az^{-1}\right)^5}$
24.	–	–	$a^k \cos k\pi$	$\frac{1}{1+az^{-1}}$

$x(t) = 0$ za $t < 0$

$x(kT) = x[k] = 0$ za $k < 0$

Ako nije drugačije naglašeno: $k = 0, 1, 2, 3, \ldots$

Važna svojstva i teoreme Z-transformacije

	$x(t)$ or $x(k)$	$\mathcal{Z}\{x(t)\}$ or $\mathcal{Z}\{x(k)\}$
1.	$ax(t)$	$aX(z)$
2.	$ax_1(t)+bx_2(t)$	$aX_1(z)+bX_2(z)$
3.	$x(t+T)$ or $x(k+1)$	$zX(z)-zx(0)$
4.	$x(t+2T)$	$z^2X(z)-z^2x(0)-zx(T)$
5.	$x(k+2)$	$z^2X(z)-z^2x(0)-zx(1)$
6.	$x(t+kT)$	$z^kX(z)-z^kx(0)-z^{k-1}x(T)-\ldots-zx(kT-T)$
7.	$x(t-kT)$	$z^{-k}X(z)$
8.	$x(n+k)$	$z^kX(z)-z^kx(0)-z^{k-1}x(1)-\ldots-zx(k1-1)$
9.	$x(n-k)$	$z^{-k}X(z)$
10.	$tx(t)$	$-Tz\frac{d}{dz}X(z)$
11.	$kx(k)$	$-z\frac{d}{dz}X(z)$
12.	$e^{-at}x(t)$	$X(ze^{aT})$
13.	$e^{-ak}x(k)$	$X(ze^{a})$
14.	$a^kx(k)$	$X\left(\frac{z}{a}\right)$
15.	$ka^kx(k)$	$-z\frac{d}{dz}X\left(\frac{z}{a}\right)$
16.	$x(0)$	$\lim_{z\to\infty} X(z)$ if the limit exists
17.	$x(\infty)$	$\lim_{z\to 1}\left[\left(1-z^{-1}\right)X(z)\right]$ if $\left(1-z^{-1}\right)X(z)$ is analytic on and outside the unit circle
18.	$\nabla x(k)=x(k)-x(k-1)$	$\left(1-z^{-1}\right)X(z)$
19.	$\Delta x(k)=x(k+1)-x(k)$	$(z-1)X(z)-zx(0)$
20.	$\sum_{k=0}^{n}x(k)$	$\frac{1}{1-z^{-1}}X(z)$
21.	$\frac{\partial}{\partial a}x(t,a)$	$\frac{\partial}{\partial a}X(z,a)$
22.	$k^mx(k)$	$\left(-z\frac{d}{dz}\right)^mX(z)$
23.	$\sum_{k=0}^{n}x(kT)y(nT-kT)$	$X(z)Y(z)$
24.	$\sum_{k=0}^{\infty}x(k)$	$X(1)$