



# DIGITALNA OBRAĐA SLIKE

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ČAS 7 – DETEKCIJA IVICA, LINIJA, ČOŠKOVA; SEGMENTACIJA SLIKE

# Kako detektovati ivice?

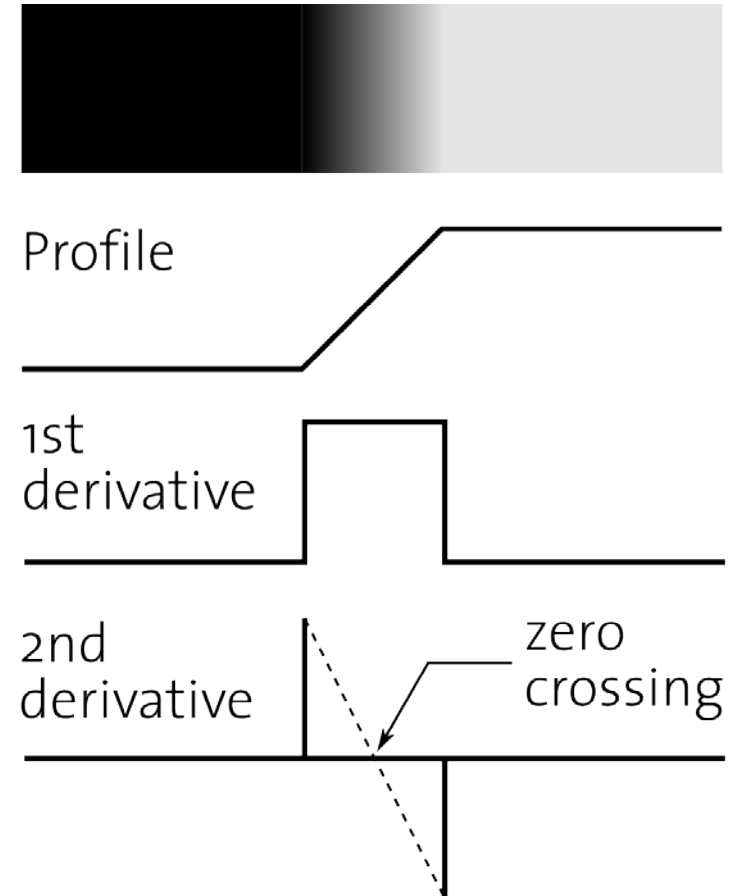
Ideja:

Nagle promene intenziteta u slici se mogu detektovati diferencijalnim operatorima prvog i drugog reda

$$\frac{\partial f}{\partial x} = f'(x) = f(x+1) - f(x)$$

$$\begin{aligned} \frac{\partial^2 f}{\partial x^2} &= \frac{\partial f'(x)}{\partial x} = f'(x+1) - f'(x) = \\ &= f(x+2) - f(x+1) - f(x+1) + f(x) = \\ &= f(x+2) - 2f(x+1) + f(x) \end{aligned}$$

$$\frac{\partial^2 f}{\partial x^2} = f''(x) = f(x+1) - 2f(x) + f(x-1)$$



# Osnovni koraci pri detekciji ivica

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## 1) Niskofrekventno filtriranje

Potrebno je kako bi se potisnuo šum i nerelevantni sitni detalji

## 2) Detekcija ivičnih tačaka

Bazira se na pronalaženju tačaka u čijoj okolini postoji značajna promena intenziteta. U ovom koraku se izdvajaju tačke koje potencijalno pripadaju ivici

## 3) Lokalizacija ivice

Od svih kandidata za ivične piksele biraju se samo oni koji najverovatnije pripadaju ivici

# Detekcija ivica bazirana na gradijentu

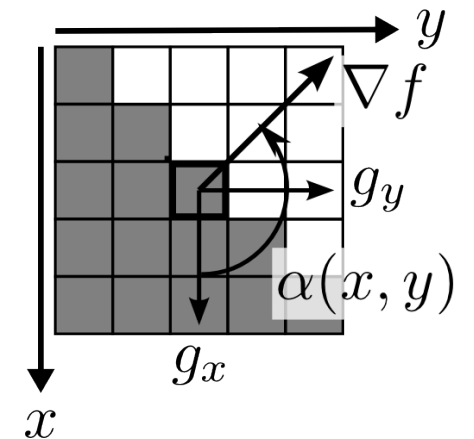
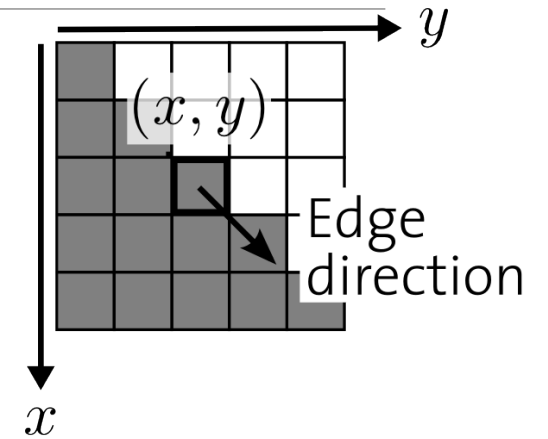
Gradijent slike sadrži informaciju o pravcu i amplitudi maksimalne promene intenziteta za svaki piksel u slici pa se stoga može iskoristiti za detekciju ivica

$$G[f(x, y)] = \nabla f(x, y) = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = (f_x, f_y)$$

$$\theta = \arctan \left( \frac{f_y}{f_x} \right)$$

$$|\nabla f| = \sqrt{f_x^2 + f_y^2}$$

$$|\nabla f| = |f_x| + |f_y|$$



# Najčešći operatori bazirani na gradijentu

**Robertsov  
operator**

$$h_x[m, n] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$h_y[m, n] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

**Pruit operator**

$$h_x[m, n] = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$
$$h_y[m, n] = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

**Sobelov  
operator**

$$h_x[m, n] = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$
$$h_y[m, n] = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

# Sobelov operator - primer

```
I = imread('building.tif');
I = double(I)/255;

Hx = [-1 -2 -1; 0 0 0; 1 2 1]
Hy = Hx'

Gx = imfilter(I, Hx, 'replicate', 'same');
Gy = imfilter(I, Hy, 'replicate', 'same');

figure('Name', 'Vertikalni gradijent'); imshow(Gx, []);
figure('Name', 'Horizontalni gradijent'); imshow(Gy, []);

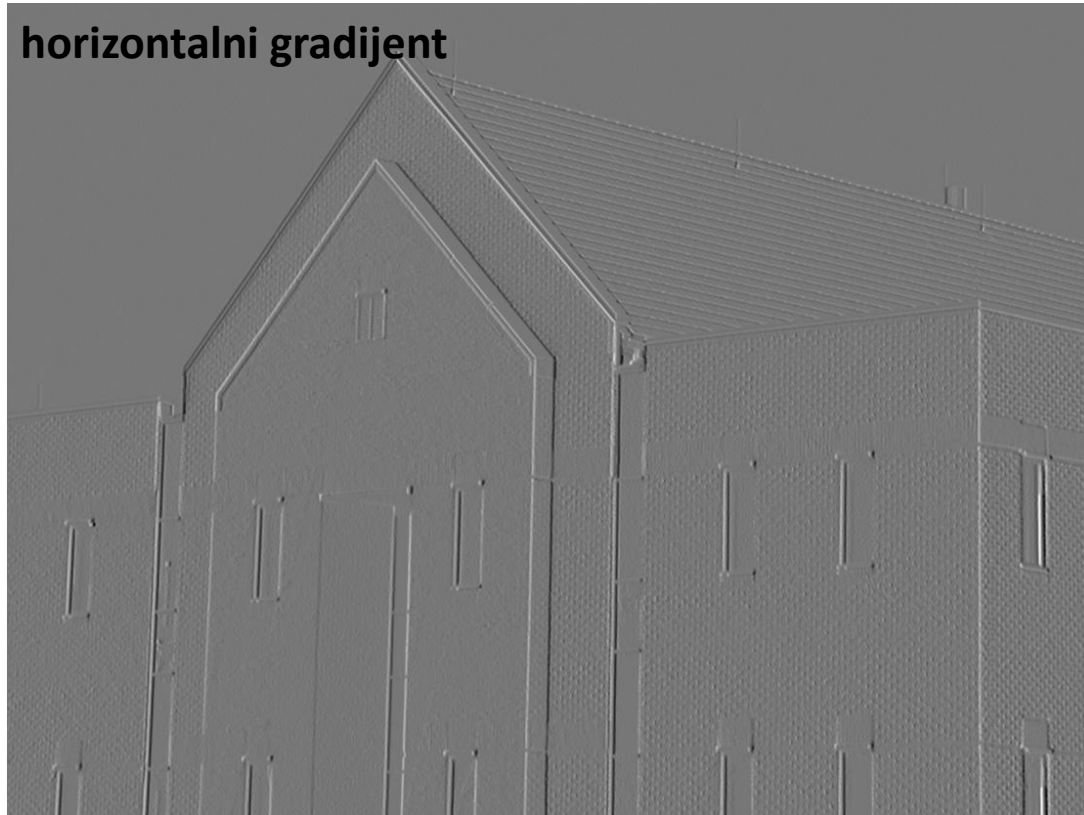
Gm = (Gx.^2 + Gy.^2).^0.5;
Ga = atan(Gy./Gx);
Ga(Gx == 0) = 0;

figure('Name', 'Magnituda gradijenta'); imshow(Gm, []);
figure('Name', 'Ugao gradijenta'); imagesc(Ga);
```

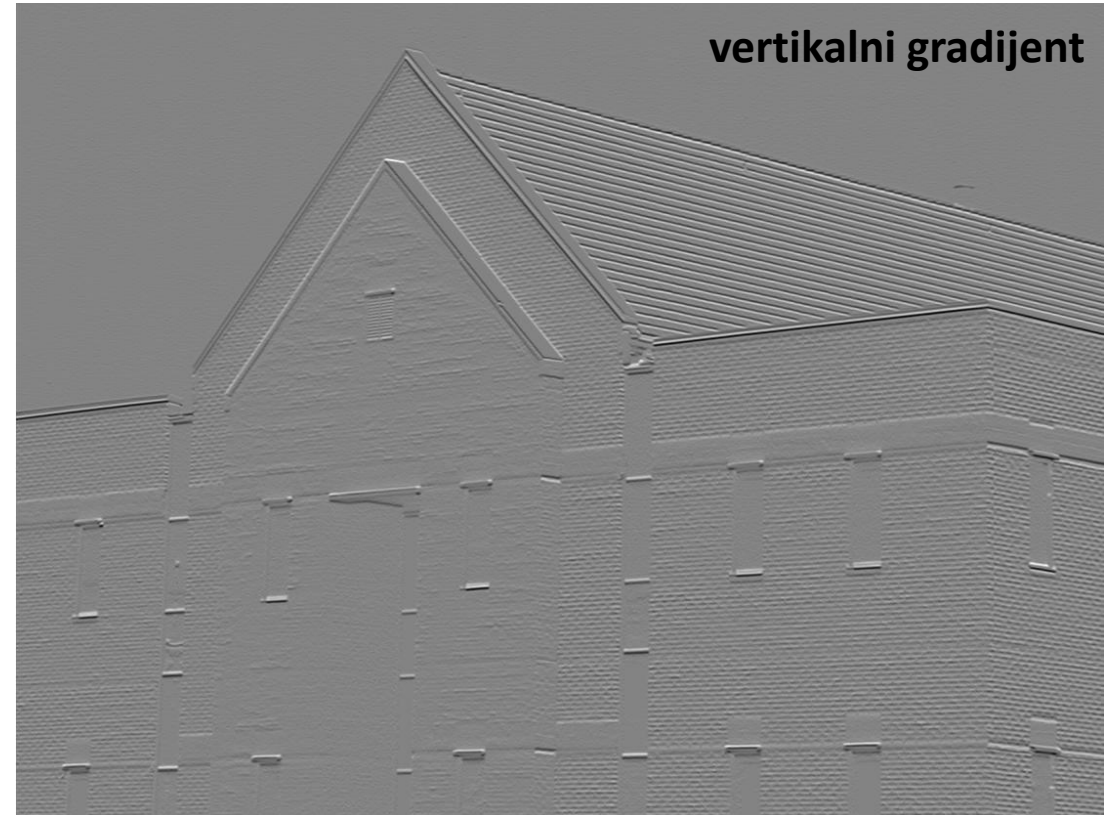
# Sobelov operator - primer



horizontalni gradijent



vertikalni gradijent

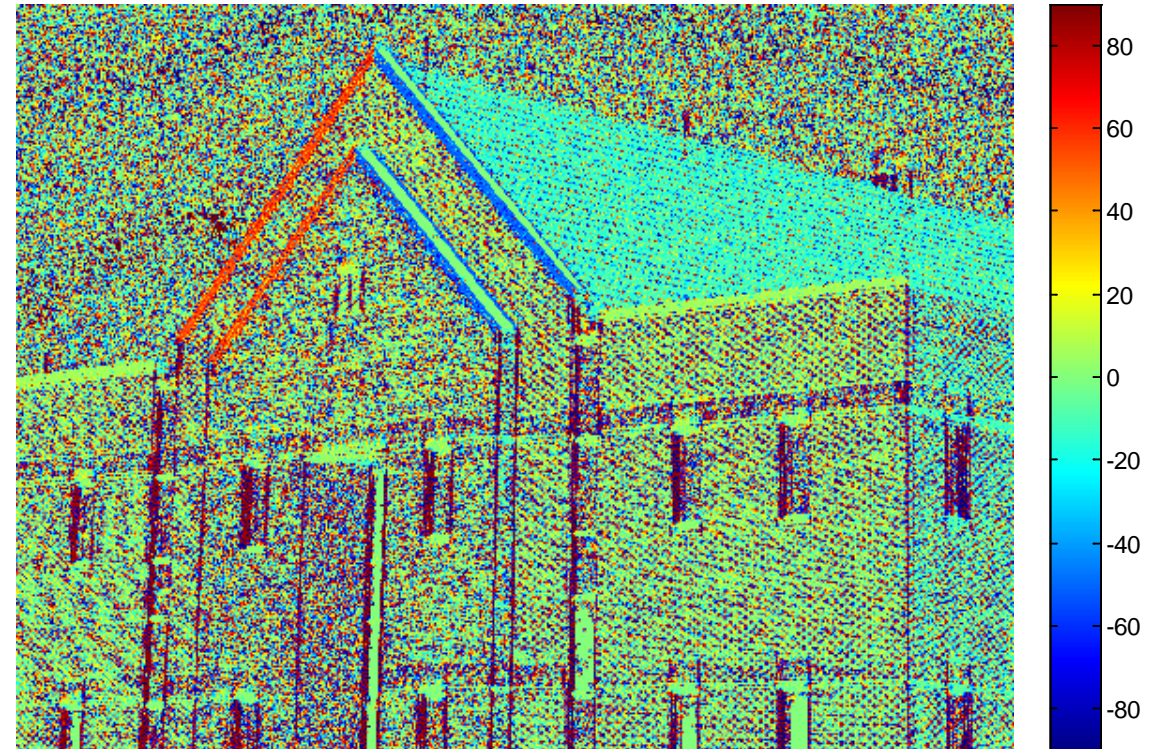


# Sobelov operator - primer

magnituda gradijenta



ugao gradijenta



# Kombinacija sa NF filtrom

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magnituda gradijenta



magnituda gradijenta slike isfiltrirane  
Gausovim filtrom 9x9,  $\sigma=5$



# Detekcija ivica na bazi gradijenta

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Ivični piksel predstavlja onaj piksel kod koga magnituda gradijenta ima značajnu vrednost

Prema tome detekcija ivica se može obaviti poređenjem magnitude gradijenta sa pragom. Pikseli čija magnituda gradijenta prelazi prag bivaju označeni kao ivični pikseli.

$$E[m, n] = \begin{cases} 1, & |\nabla f[m, n]| \geq T \\ 0, & |\nabla f[m, n]| < T \end{cases}$$

# Poređenje gradijenta s pragom

```
Gx = imfilter(I, Hx, 'replicate', 'same');
Gy = imfilter(I, Hy, 'replicate', 'same');
Gm_filt = (Gx.^2 + Gy.^2).^0.5;

thr1 = 0.1*max(Gm(:));
thr2 = 0.3*max(Gm(:));
thr3 = 0.5*max(Gm(:));
E = zeros(size(Gm));
E(Gm>thr1) = 1;
figure('Name', 'THR=0.1*MAX'); imshow(E, []);
E = zeros(size(Gm));
E(Gm>thr2) = 1;
figure('Name', 'THR=0.3*MAX'); imshow(E, []);
E = zeros(size(Gm));
E(Gm>thr3) = 1;
figure('Name', 'THR=0.5*MAX'); imshow(E, []);
```

```
Ifilt = imfilter(I, fspecial('gaussian', [9 9], 5),
'replicate', 'same');
Gx = imfilter(I, Hx, 'replicate', 'same');
Gy = imfilter(I, Hy, 'replicate', 'same');
Gm_filt = (Gx.^2 + Gy.^2).^0.5;

thr1 = 0.1*max(Gm_filt(:));
thr2 = 0.3*max(Gm_filt(:));
thr3 = 0.5*max(Gm_filt(:));
E = zeros(size(Gm_filt));
E(Gm_filt>thr1) = 1;
figure('Name', 'FILT THR=0.1*MAX'); imshow(E, []);
E = zeros(size(Gm_filt));
E(Gm_filt>thr2) = 1;
figure('Name', 'FILT THR=0.3*MAX'); imshow(E, []);
E = zeros(size(Gm_filt));
E(Gm_filt>thr3) = 1;
figure('Name', 'FILT THR=0.5*MAX'); imshow(E, []);
```

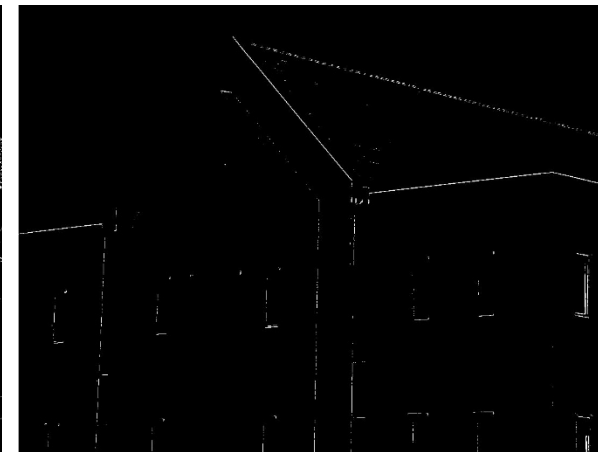
# Poređenje gradijenta s pragom

THR = 0.1·MAX

THR = 0.3·MAX

THR = 0.5·MAX

bez filtriranja



sa filtriranjem



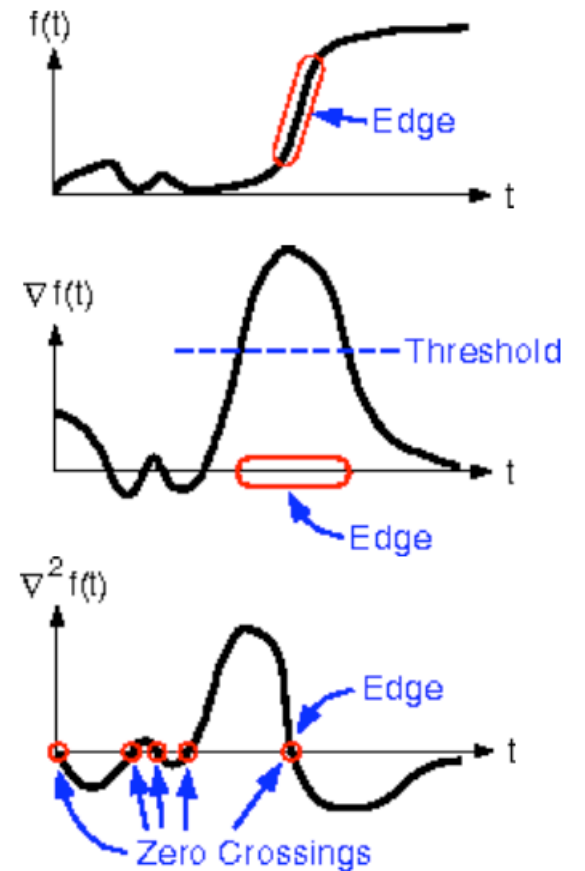
# Laplasov operator

Gradijentni operatori imaju tendenciju da daju široke ivice – lošija lokalizacija

Postepeni prelazi će rezultovati malim gradijentom i verovatno neće biti detektovani

Drugi izvod daje bolju lokalizaciju ivica!

Položaj ivice se određuje prolascima kroz nulu drugog izvoda slike



# Laplasov operator

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Detekcija ivica Laplasovim operatorom se izvodi tako što se najpre odredi laplasijan slike:

$$\nabla^2 f(x, y) = \frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2}$$

a potom se detektuju značajne vrednosti laplasijana i za njih odrede prolasci kroz nulu.

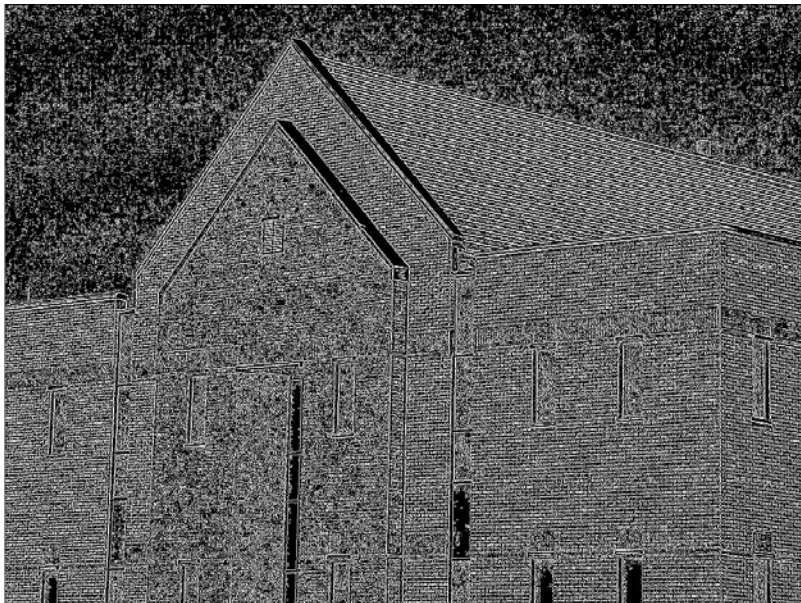
**Diskretne  
aproskimacije  
laplasijana:**

$$h_L[m, n] = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \quad h_L[m, n] = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$
$$h_L[m, n] = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix} \quad h_L[m, n] = \begin{bmatrix} -2 & 1 & -2 \\ 1 & 4 & 1 \\ -2 & 1 & -2 \end{bmatrix}$$

# Laplasov operator - primer

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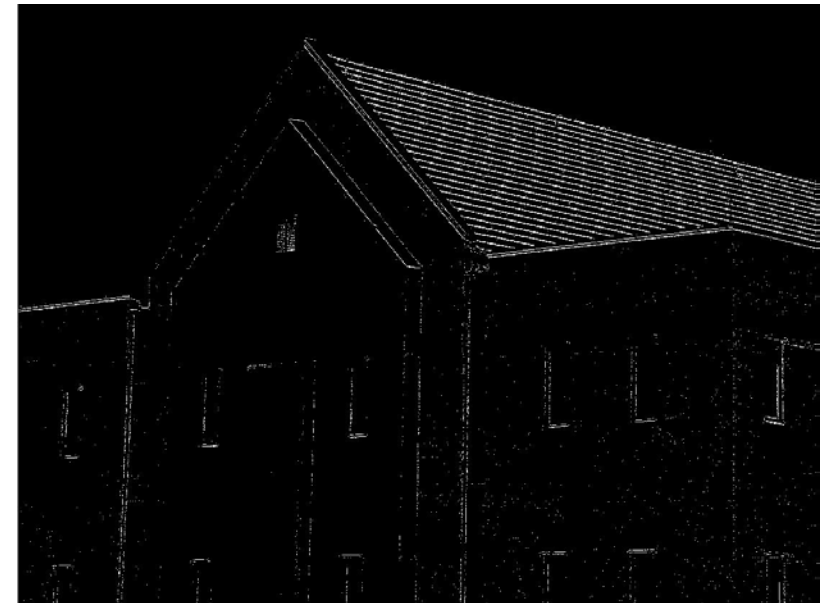
**THR = 0.1**



**THR = 0.4**



**THR = 0.8**



# Laplasov operator

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Dosta osetljiviji na šum od operatora baziranih na gradijentu

Detekcija nultih preseka rezultuje tankim ivicama širine 1 piksel

Laplasijan je izotropan tako da ne daje informaciju o smeru ivice

# LoG (Laplacian of Gaussian) operator

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Osetljivost na šum se smanjuje prethodnim niskofrekventnim filtriranjem

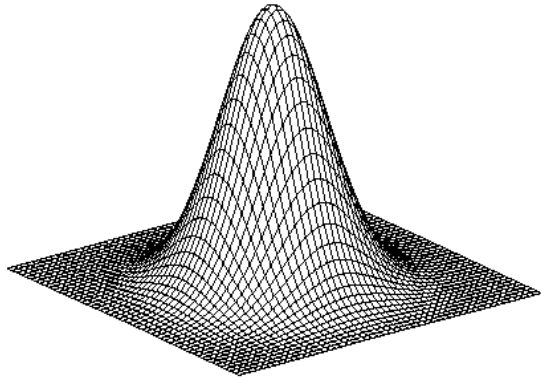
Može se spojiti u jedan korak primenom laplasijana na impulsi odziv NF filtra. Ako je NF filter Gausov onda se takav operator naziva LoG (Laplacian of Gaussian)

$$h_{NF}(x, y) = \left[ \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}} \right] \left[ \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} \right]$$

$$h_{LoG}(x, y) = \frac{1}{\pi\sigma^4} \left( 1 - \frac{x^2 + y^2}{2\sigma^2} \right) e^{-\frac{x^2 + y^2}{2\sigma^2}}$$

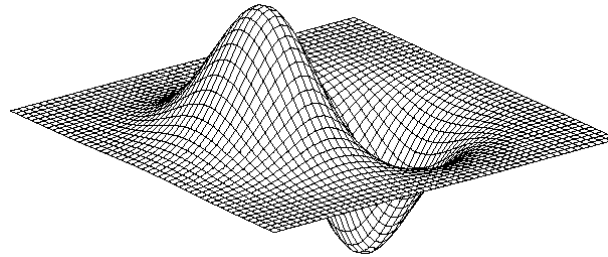
# Operatori izvedeni iz Gausove funkcije

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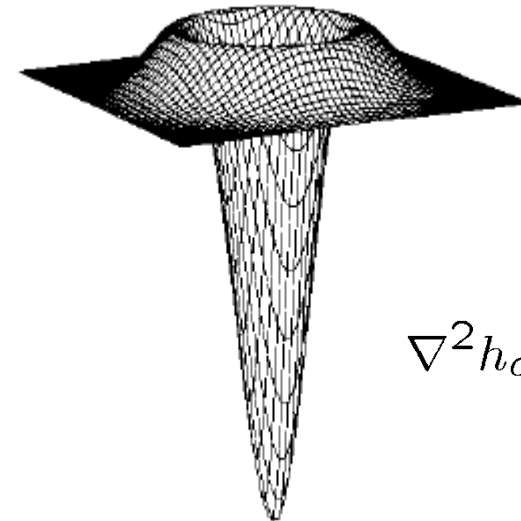
$$h_{\sigma}(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

**Gaussian**



$$\frac{\partial}{\partial x} h_{\sigma}(u, v)$$

**DroG – Derivative of  
Gaussian**



$$\nabla^2 h_{\sigma}(u, v)$$

**LoG – Laplacian of Gaussian  
*Mexican Hat (Sombrero)***

# Kanijev (Canny) algoritam za detekciju ivica

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Nastao 1986 na MIT-u

I dalje ima superiorne karakteristike u pogledu detekcije ivica

Kani je formulisao tri uslova koja bi idealni detektor trebalo da ispuni:

- 1) Mali procenat lažnih detekcija, visok procenat pravih detekcija
- 2) Dobra lokalizacija
- 3) Jedinični odziv na ivicu (sve ivice su širine 1 piksel)

Kani je matematički opisao postavljene kriterijume i pokušao da nađe optimalno rešenje

U opštem slučaju analitičko rešenje ovog optimizacionog problema ne postoji ali se on može rešiti numeričkim metodama

# Kanijev algoritam za detekciju ivica

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Numeričko rešenje Kanijevog problema dovodi do toga da jedobra aproksimacija optimalnog detektora idealnih ivica za 1-D slučaj ivica zašumljenih Gausovim šumom prvi izvod Gausijana:

$$\frac{d}{dx} e^{-\frac{x^2}{2\sigma^2}} = -\frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}$$

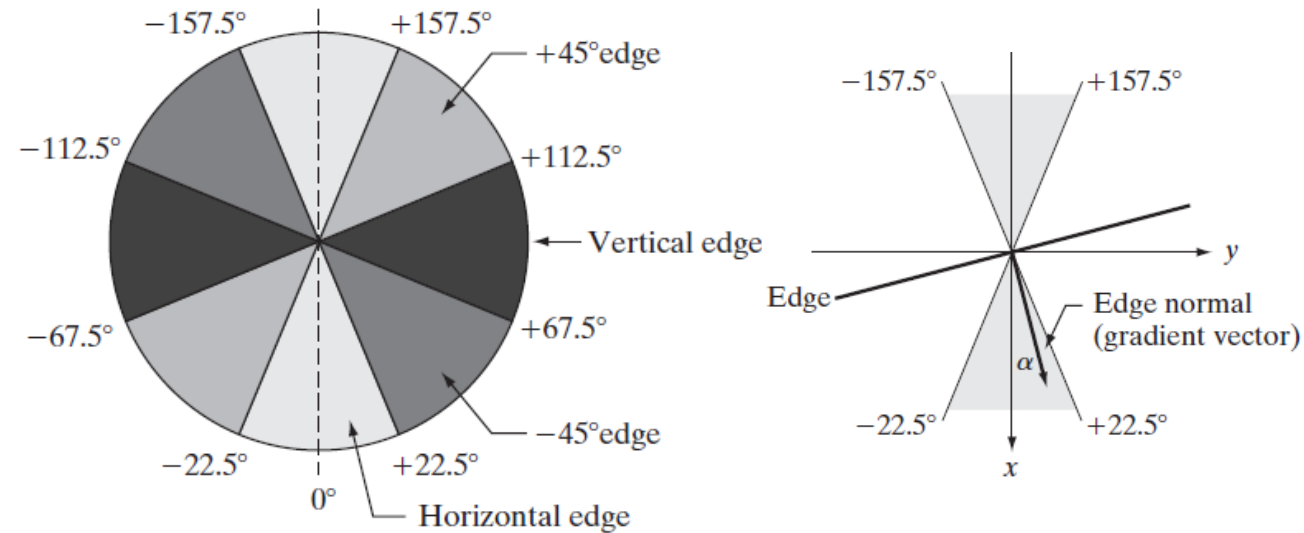
U 2-D slučaju detekcija se radi u smeru dominantnog gradijenta (normala na ivicu)

# Kanijev algoritam za detekciju ivica

Najpre se primeni odgovarajući operator za detekciju gradijenta. U Kanijevom algoritmu se koristi DroG operator.

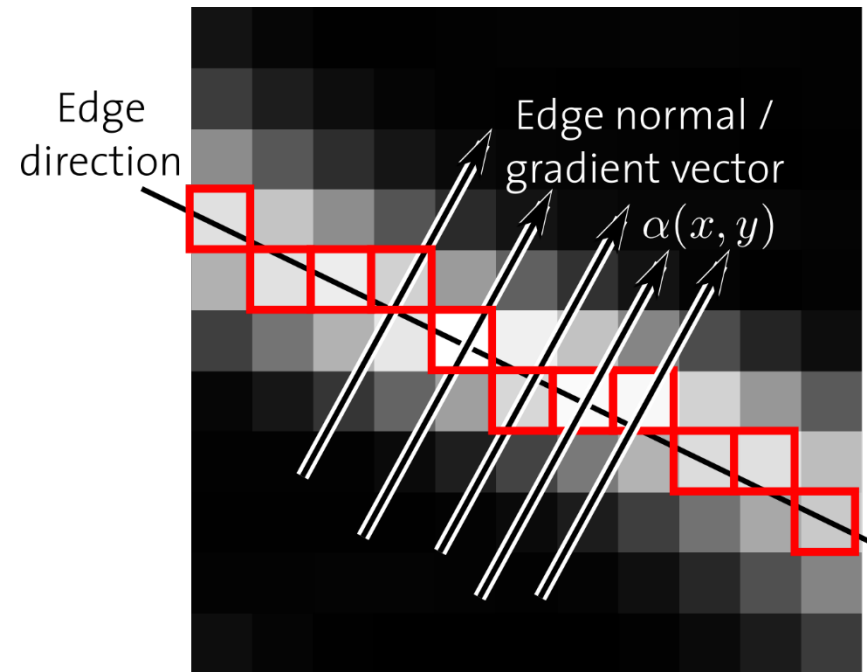
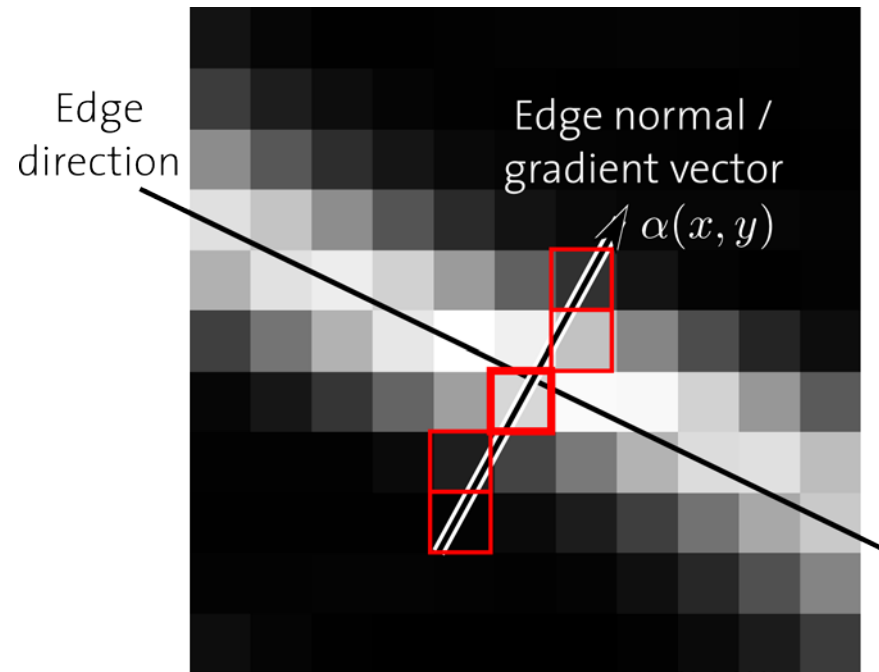
Podešavanjem  $\sigma$  Gausove funkcije postiže se kompromis između lokalizacije i osetljivosti na šum.

Sledeći korak predstavlja potiskivanje lokalnih ne-maksimuma.



horizontalna	-45° dijagonalna	vertikalna	+45° dijagonalna
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

# Potiskivanje lokalnih ne-maksimuma



# Histerezijsno poređenje s pragom

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Poređenje s pragom se obavlja u dva koraka kako bi se obezbedila što bolja robusnost na šum

Zadaju se dve vrednosti praga. Svi pikseli čija magnituda gradijenta prelazi viši prag predstavljaju sigurne ivice, dok pikseli čija magnituda gradijenta prelazi samo niži prag predstavljaju potencijalne ivice

U narednom koraku, zadržavaju se samo one potencijalne ivice koje su povezane sa nekom sigurnom ivicom

# Podrška u MATLAB-u

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**binarna\_matrica\_ivica = edge(ulazna\_slika, metod, opcije)**

```
help edge
```

```
Es = edge(I, 'sobel', 'nothinning');  
figure('Name', 'Sobel operator'); imshow(Es, []);
```

```
E1 = edge(I, 'log', 0.0006, 4);  
figure('Name', 'LoG operator'); imshow(E1, []);
```

```
Ec = edge(I, 'canny', 0.2, 2);  
figure('Name', 'Canny operator'); imshow(Ec, []);
```

**Isprobati funkciju edge za različite vrednosti parametara.**

# Primer

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ulaz



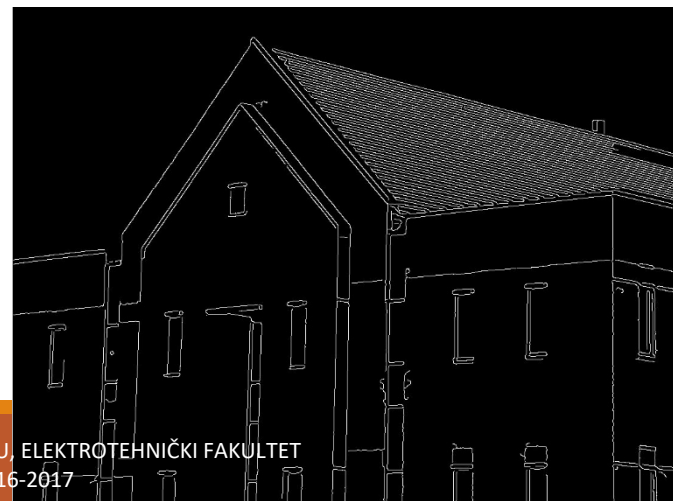
Sobel



LoG



Canny



# Harisov detektor – detekcija čoškova

---

$$A(x, y) = \left[ \nabla_x I(x, y) \right]^2$$

$$B(x, y) = \left[ \nabla_y I(x, y) \right]^2$$

$$C(x, y) = \nabla_x I(x, y) \cdot \nabla_y I(x, y)$$

$$M(x, y) = \begin{bmatrix} A & C \\ C & B \end{bmatrix}$$

$$\bar{A}(x, y) = H_\sigma * A(x, y)$$

$$\bar{B}(x, y) = H_\sigma * B(x, y)$$

$$\bar{C}(x, y) = H_\sigma * C(x, y)$$

$$\bar{M}(x, y) = \begin{bmatrix} \bar{A} & \bar{C} \\ \bar{C} & \bar{B} \end{bmatrix}$$

$$R = \det \bar{M} - k \left( \text{tr} \bar{M} \right)^2$$

$$R = \bar{A}\bar{B} - \bar{C}^2 - k \left( \bar{A} + \bar{B} \right)^2$$

# Harisov detektor – detekcija ćoškova

```
I = rgb2gray(imread('shapes_gray.png'));
I = im2double(I);

Sx = [-1 -2 -1; 0 0 0; 1 2 1]./8;
Sy = [-1 0 1; -2 0 2; -1 0 1]./8;

Ix = imfilter(I, Sx, 'replicate');
Iy = imfilter(I, Sy, 'replicate');

H = fspecial('gaussian', [5 5], 1);
A = imfilter(Ix.^2, H, 'replicate');
B = imfilter(Iy.^2, H, 'replicate');
C = imfilter(Ix.*Iy, H, 'replicate');

figure; imshow(I);
figure; imshow(A, []);
figure; imshow(B, []);
figure; imshow(C, []);
```

```
k = 0.05;
R = A.*B - C.^2 - k*((A+B).^2);
Q = R./max(R(:));

thr = 0.15;

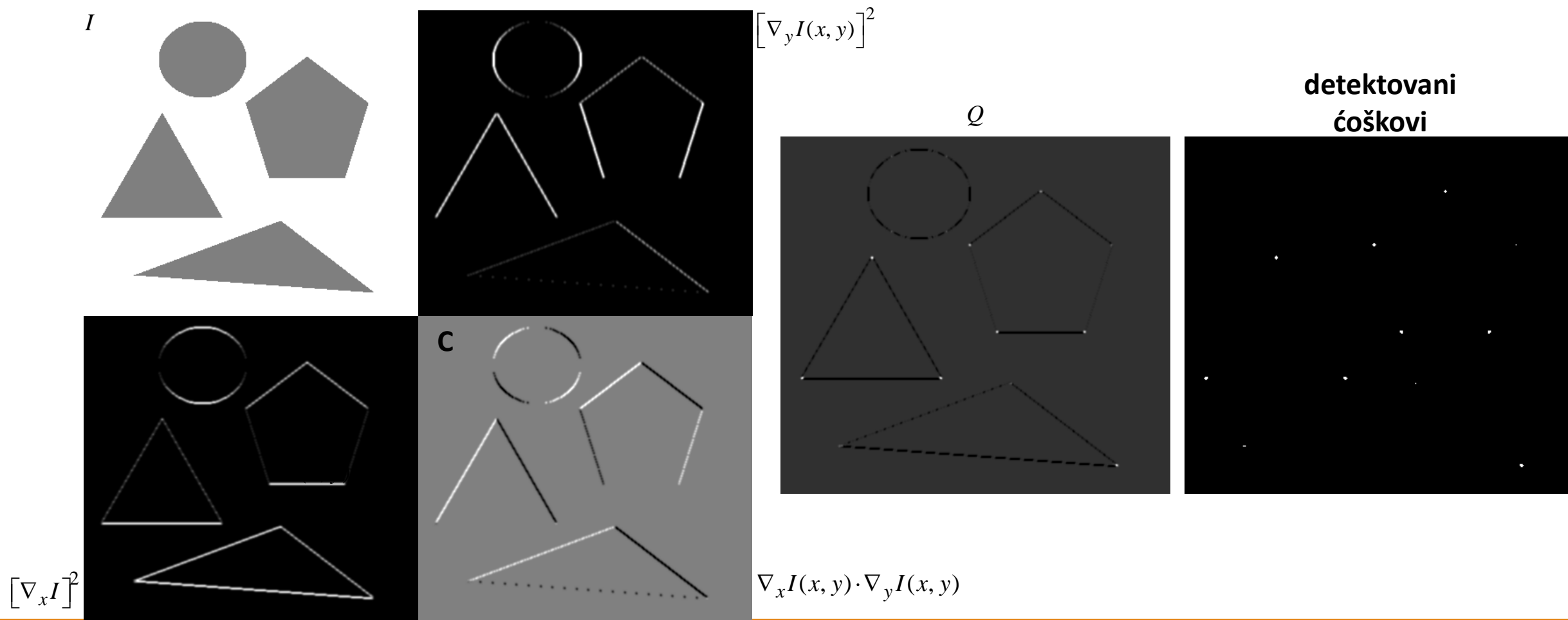
corner = Q;
corner(Q<thr) = 0;
corner(Q>=thr) = 1;

figure; imshow(corner);
```

**Probati funkciju  
za vrednosti  
praga 0.1, 0.2**

**Probati sa slikama shapes\_gray1.png i shapes.png**

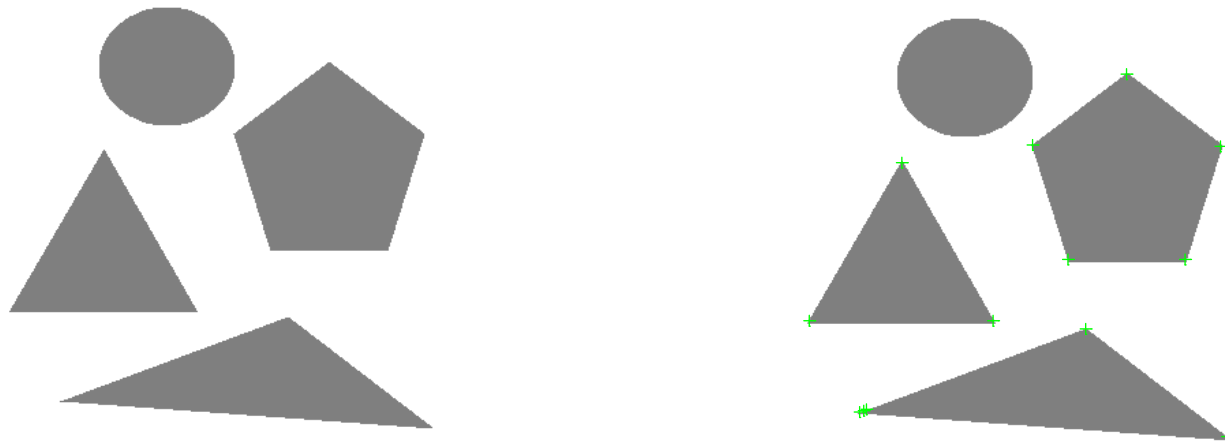
# Harisov detektor - primer



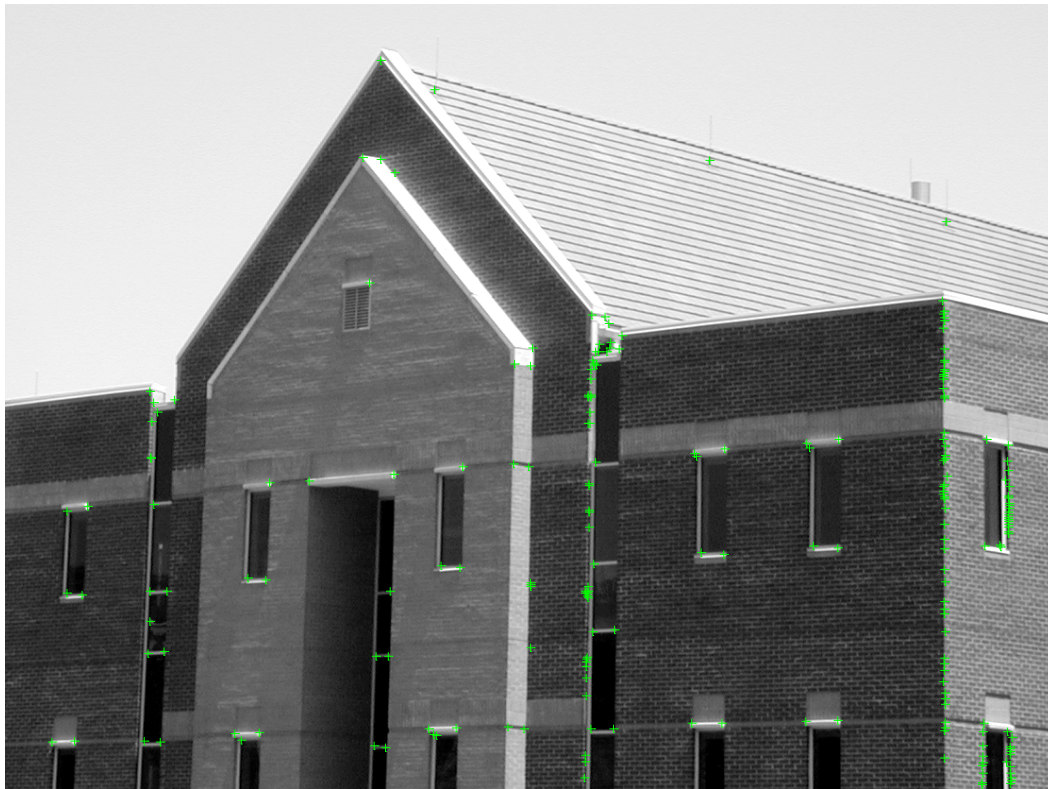
# Harisov detektor – detekcija ćoškova

```
I = rgb2gray(imread('shapes_gray.png'));  
I = im2double(I);  
  
corners = detectHarrisFeatures(I, 'MinQuality', 0.25, 'FilterSize', 5);  
figure; imshow(I); hold on; plot(corners); hold off;
```

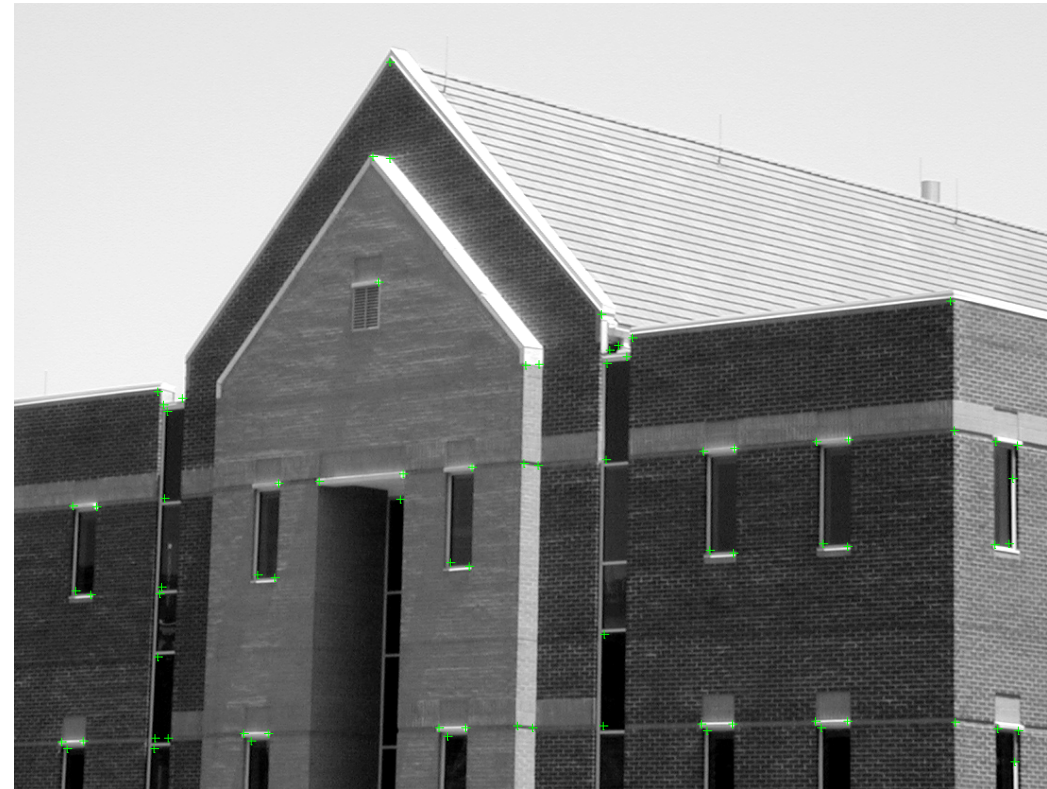
**Zbog ćega se prag toliko razlikuje ovde?**



# Harrisov detektor - primer

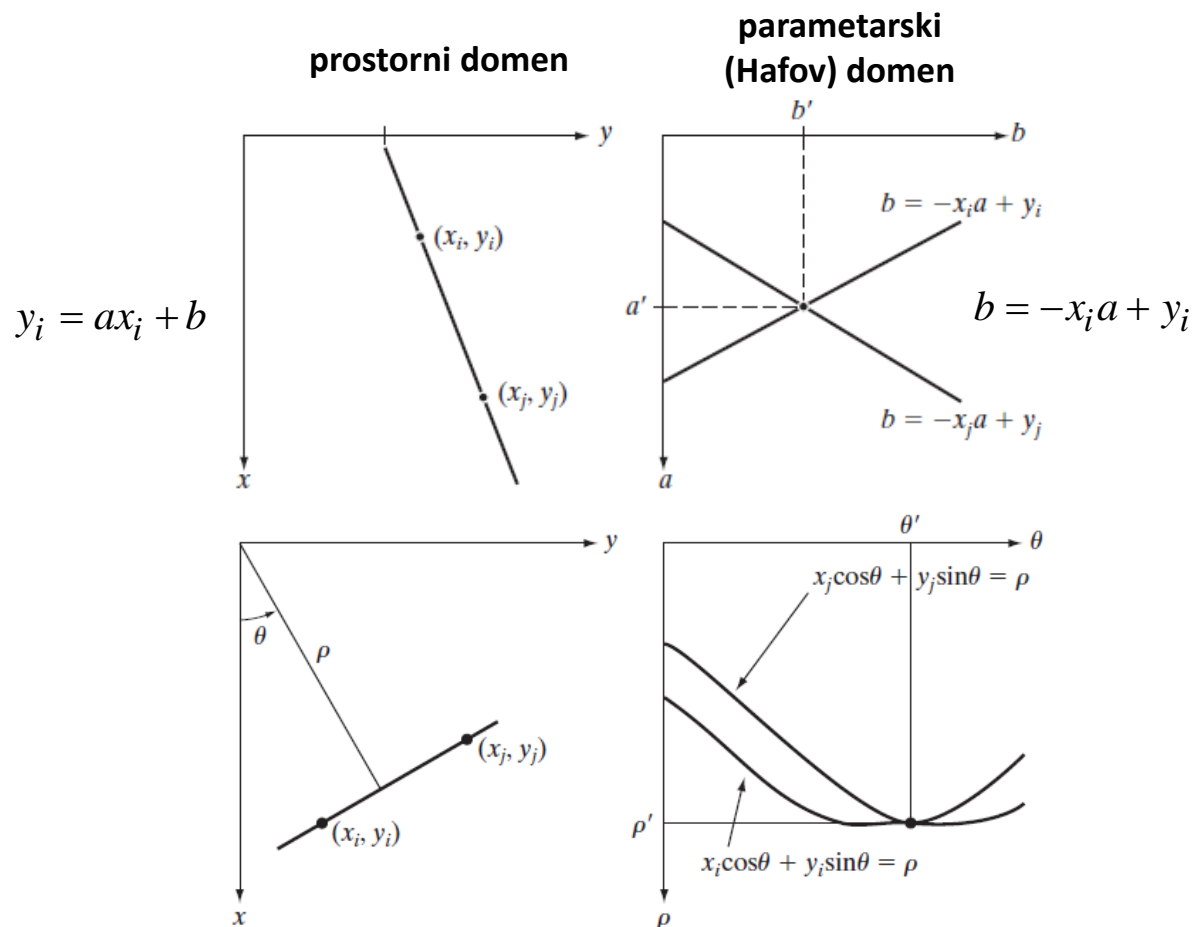


```
corners = detectHarrisFeatures(I, 'MinQuality', 0.05, 'FilterSize', 11);  
figure; imshow(I); hold on; plot(corners); hold off;
```



```
J = imfilter(I, fspecial('gaussian', [7 7], 2), 'replicate');  
corners = detectHarrisFeatures(J, 'MinQuality', 0.05, 'FilterSize', 11);  
figure; imshow(I); hold on; plot(corners); hold off;
```

# Detekcija linija – Hafova transformacija



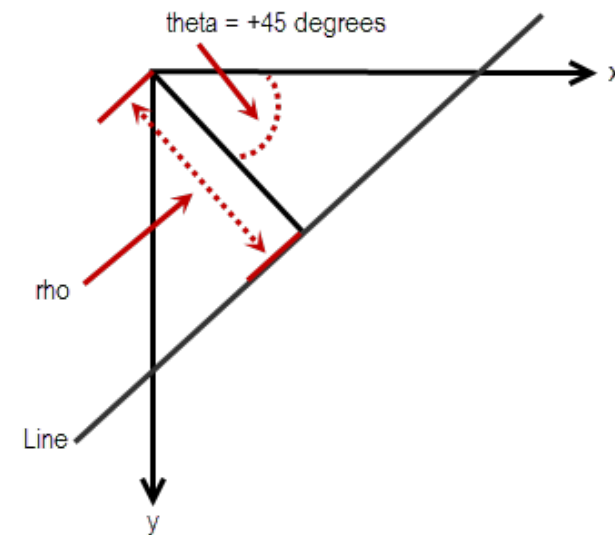
prostorni domen  
 $x \cos \theta + y \sin \theta = \rho$

parametarski domen ( $\theta, \rho$ ):

*Svaka tačka iz prostornog domena predstavlja sinusoidalnu funkciju u parametarskom domenu. Svaka prava u prostornom domenu predstavlja tačku u parametarskom domenu. Ako se tri ili više sinusoidalnih kriva seku u istoj tački parametarskom domenu to znači da su odgovarajuće taške kolinearne u prostornom domenu.*

*Ideja: Naći tačke nagomilavanja (sa najviše preseka) u parametarskom domenu.*

Funkcija hough u Matlabu za ugao normale uzima vrednosti iz opsega  $+90^\circ$  u smeru prikazanom na slici



# Hafova transformacija

```
I = rgb2gray(imread('shapes_gray.png'));
I = im2double(I);

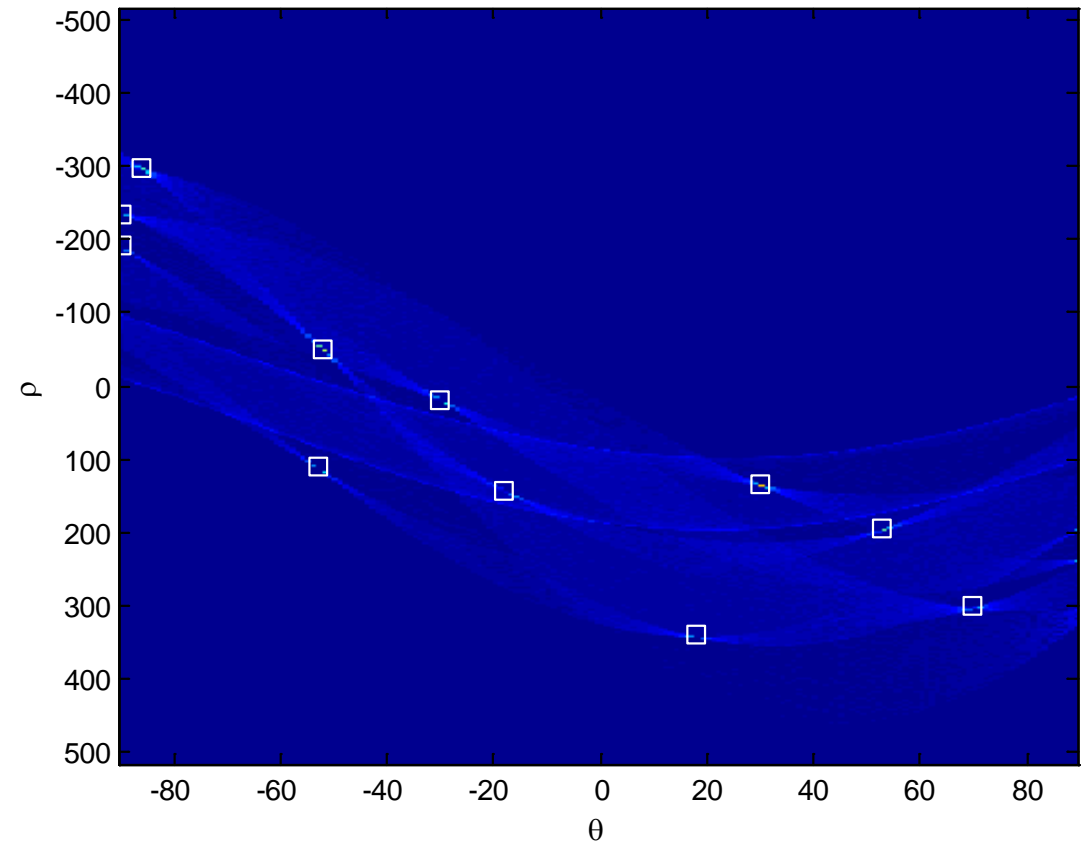
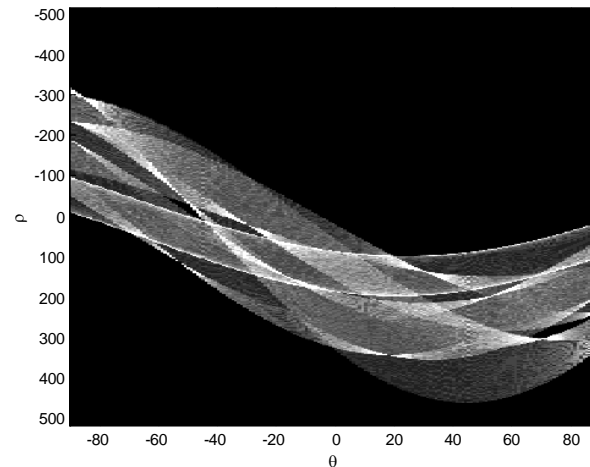
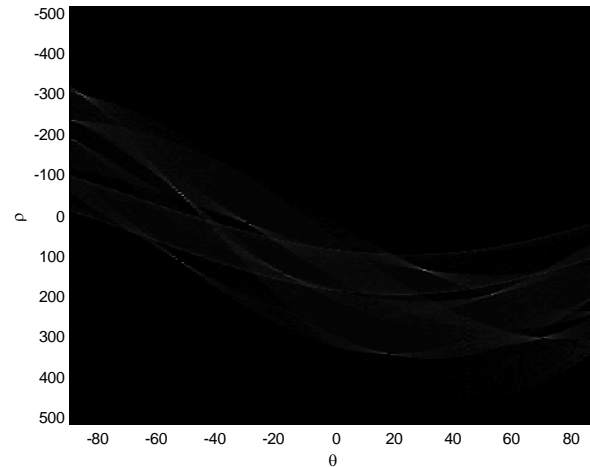
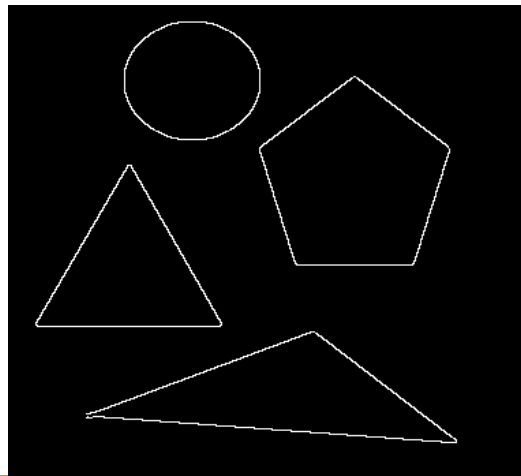
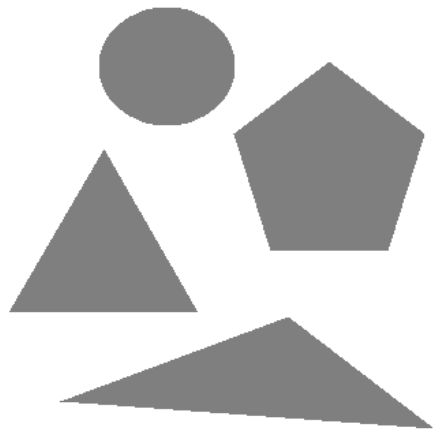
E = edge(I, 'canny');
figure; imshow(E);

[H,T,R] = hough(E);

figure; imagesc(H,'XData',T,'YData',R); colormap('gray');
xlabel('\theta'), ylabel('\rho'); axis on, axis normal;
figure; imagesc(imadjust(mat2gray(H)),'XData',T,'YData',R); colormap('gray');
xlabel('\theta'), ylabel('\rho'); axis on, axis normal;

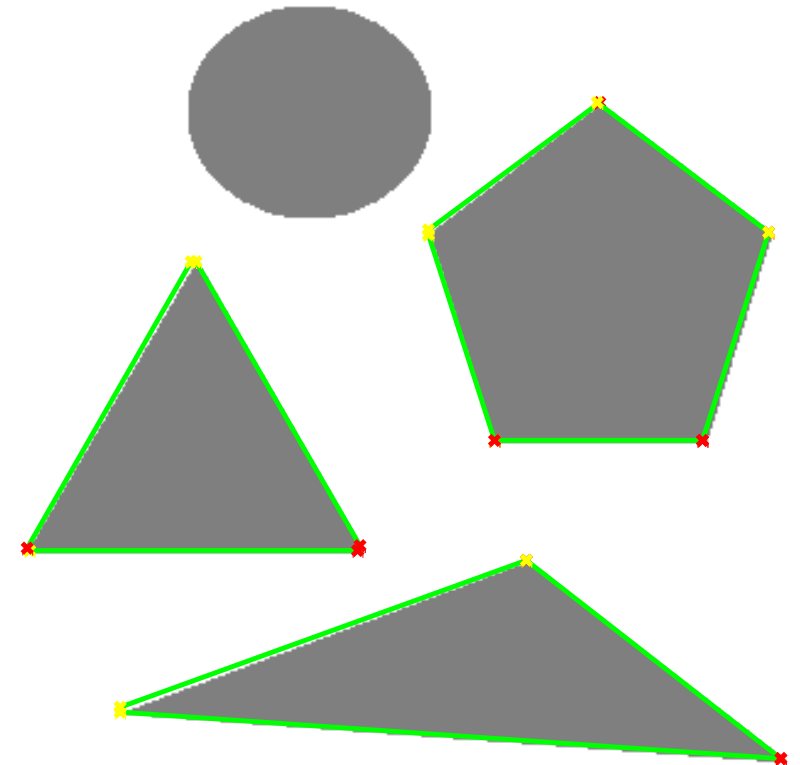
P = houghpeaks(H, 11, 'Threshold', 0.2*max(H(:)));
figure; imagesc(H, 'XData',T,'YData',R);
xlabel('\theta'), ylabel('\rho');
axis on, axis normal, hold on;
plot(T(P(:,2)),R(P(:,1)),'s','color','white');
```

# Hafova transformacija - primer



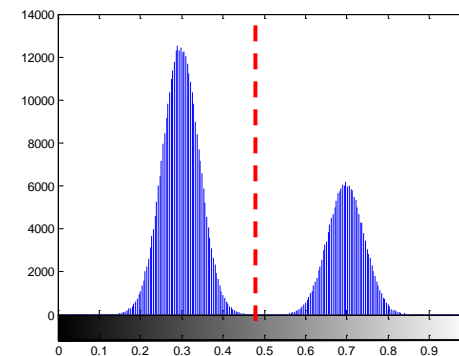
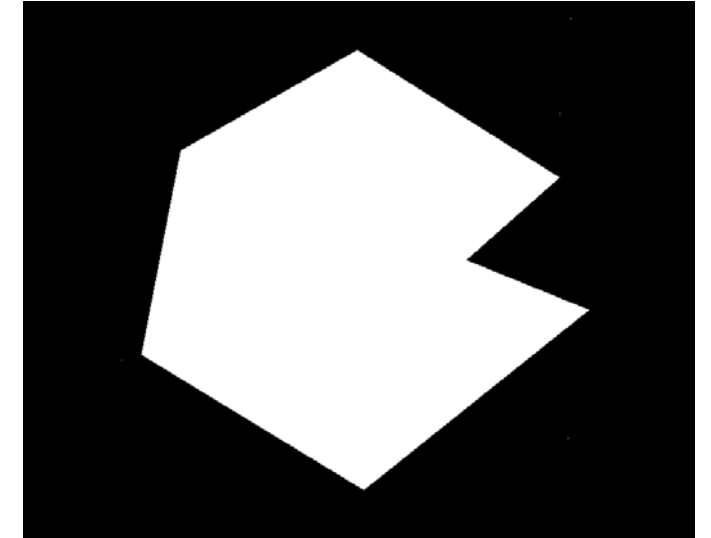
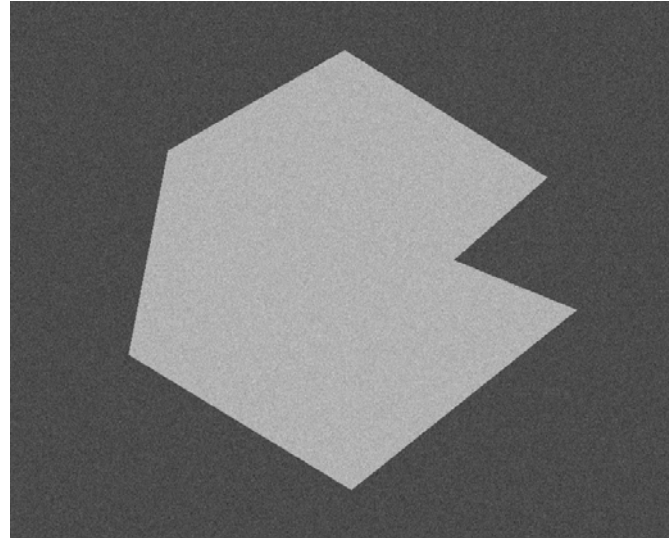
# Hafova transformacija – iscrtavanje linija

```
lines = houghlines(E,T,R,P);  
  
figure, imshow(I), hold on  
  
for k = 1:length(lines)  
    xy = [lines(k).point1; lines(k).point2];  
    plot(xy(:,1),xy(:,2),'LineWidth',2,'Color','green');  
  
    % plot beginnings and ends of lines  
    plot(xy(1,1),xy(1,2),'x','LineWidth',2,'Color','yellow');  
    plot(xy(2,1),xy(2,2),'x','LineWidth',2,'Color','red');  
end  
  
hold off;
```



# Segmentacija slike – poređenje s pragom

```
I = imread('septagon.tif');  
I = im2double(I);  
figure; imshow(I);  
figure; imhist(I); ylim('auto');  
  
[T, EM] = graythresh(I)  
  
S = I > T;  
figure; imshow(S);
```



# Segmentacija zašumljenih slika

```
I = imread('septagon_noisy.tif');
I = im2double(I);
figure; imshow(I);
figure; imhist(I); ylim('auto');

[T, EM] = graythresh(I)

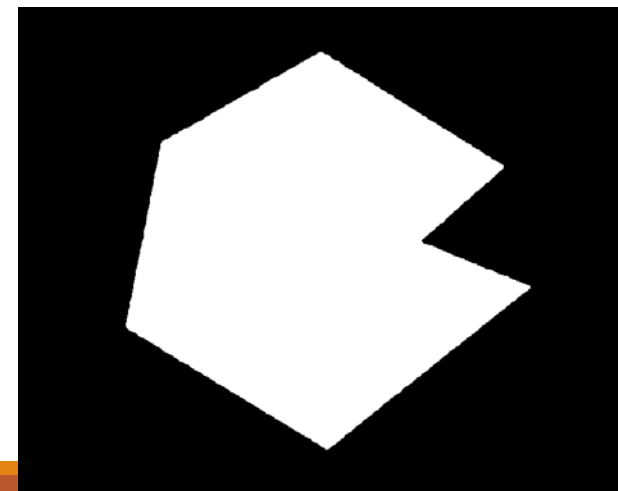
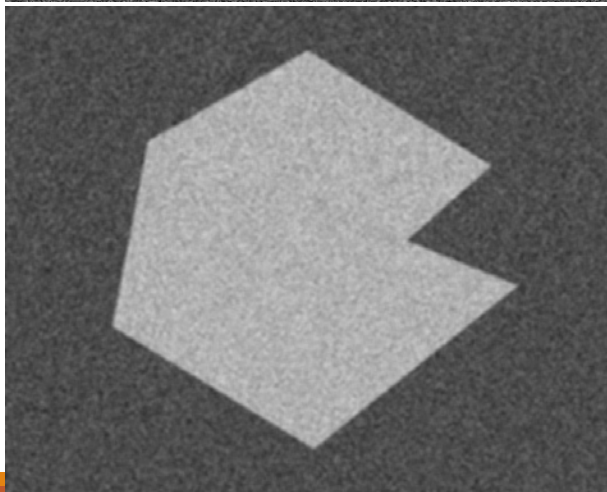
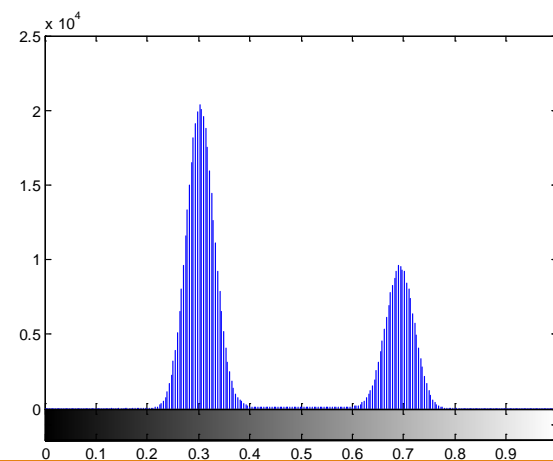
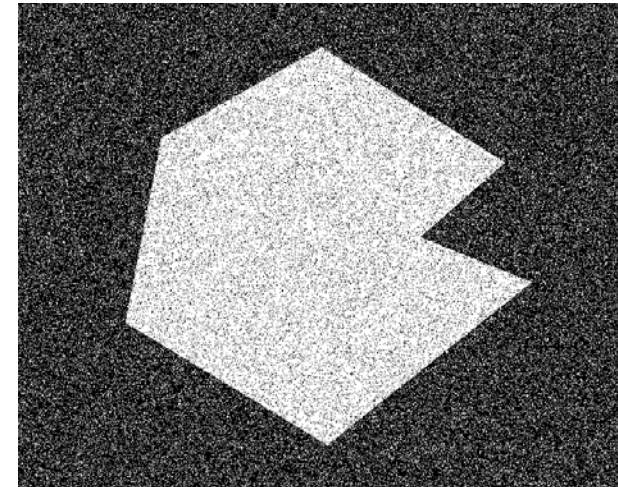
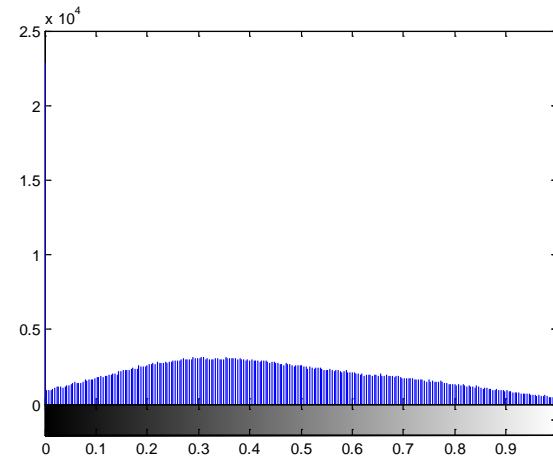
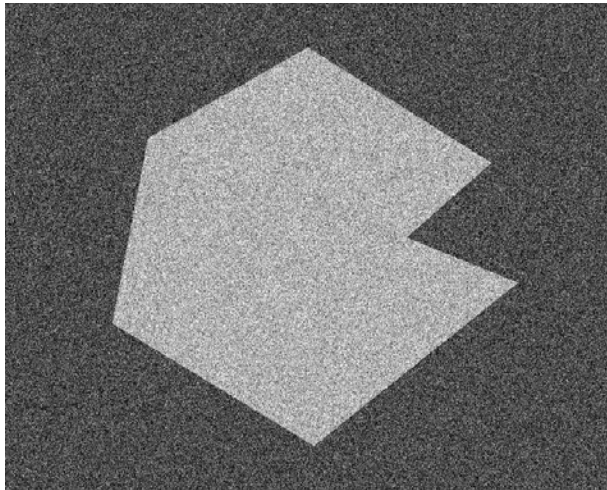
S = I > T;
figure; imshow(S);

If = imfilter(I, fspecial('gaussian', [7 7], 3), 'replicate');
figure; imshow(If);
figure; imhist(If); ylim('auto');

[T, EM] = graythresh(If)

S = If > T;
figure; imshow(S)
```

# Segmentacija zašumljenih slika



# Prostorno promenljivi prag

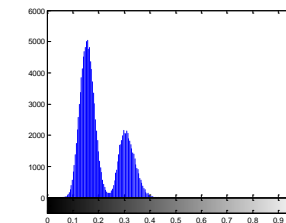
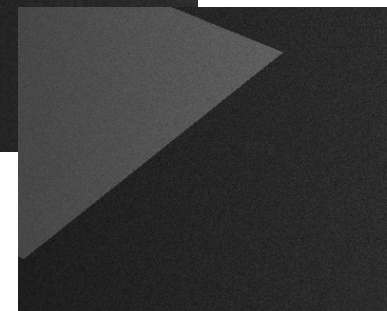
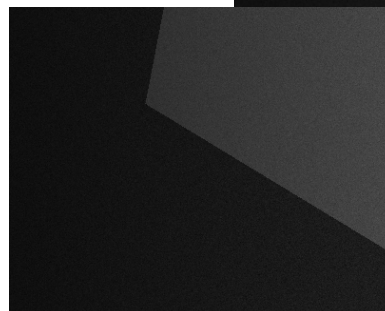
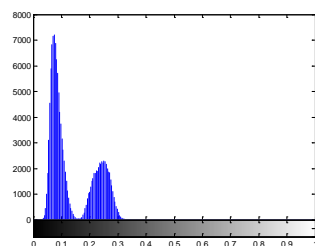
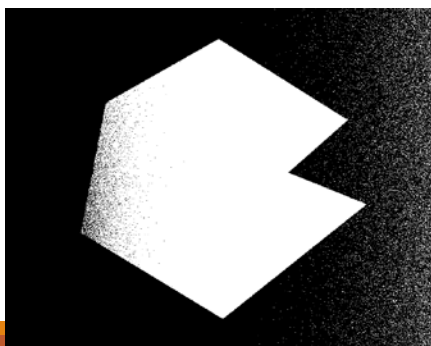
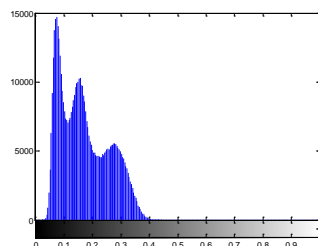
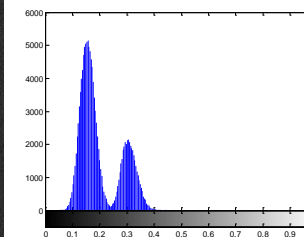
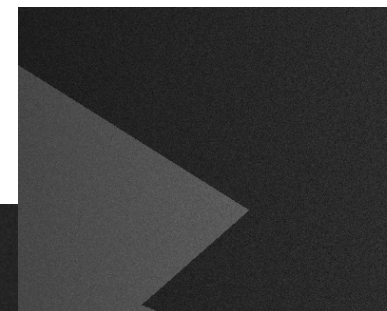
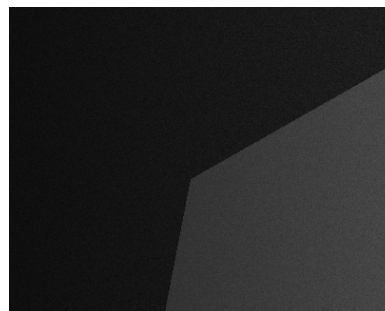
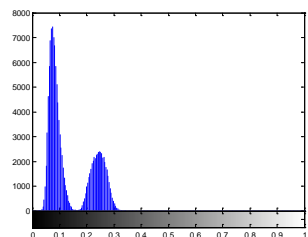
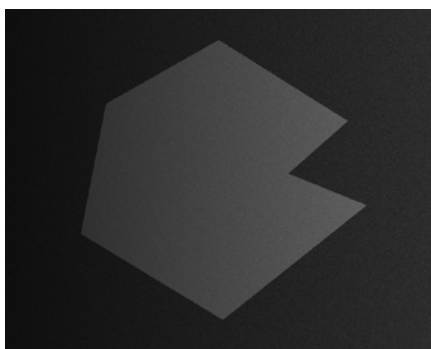
```
I = imread('septagon_noisy_shaded.tif');
I = im2double(I);
figure; imshow(I);
figure; imhist(I); ylim('auto');

[T, EM] = graythresh(I)

S = I > T;
figure; imshow(S);

I1 = I(1:round(size(I,1)/2), 1:round(size(I,2)/2));
I2 = I(1:round(size(I,1)/2), round(size(I,2)/2)+1:end);
I3 = I(round(size(I,1)/2)+1:end, 1:round(size(I,2)/2));
I4 = I(round(size(I,1)/2)+1:end, round(size(I,2)/2)+1:end);
figure; imshow(I1); figure; imhist(I1); ylim('auto');
figure; imshow(I2); figure; imhist(I2); ylim('auto');
figure; imshow(I3); figure; imhist(I3); ylim('auto');
figure; imshow(I4); figure; imhist(I4); ylim('auto');
```

# Prostorno promenljivi prag



# Prostorno promenljivi prag

```
S = zeros(size(I));

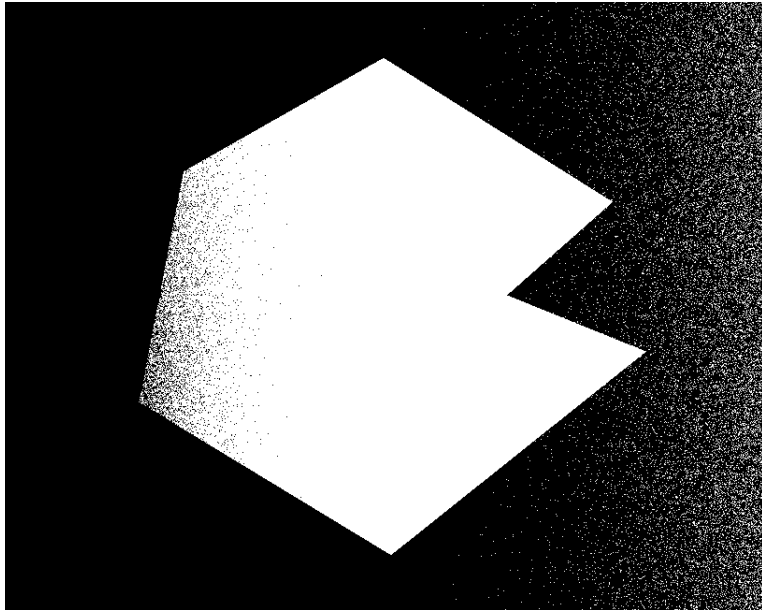
S(1:round(size(I,1)/2), 1:round(size(I,2)/2)) = I1 > graythresh(I1);
S(1:round(size(I,1)/2), round(size(I,2)/2)+1:end) = I2 > graythresh(I2);
S(round(size(I,1)/2)+1:end, 1:round(size(I,2)/2)) = I3 > graythresh(I3);
S(round(size(I,1)/2)+1:end, round(size(I,2)/2)+1:end) = I4 > graythresh(I4);

figure; imshow(S);

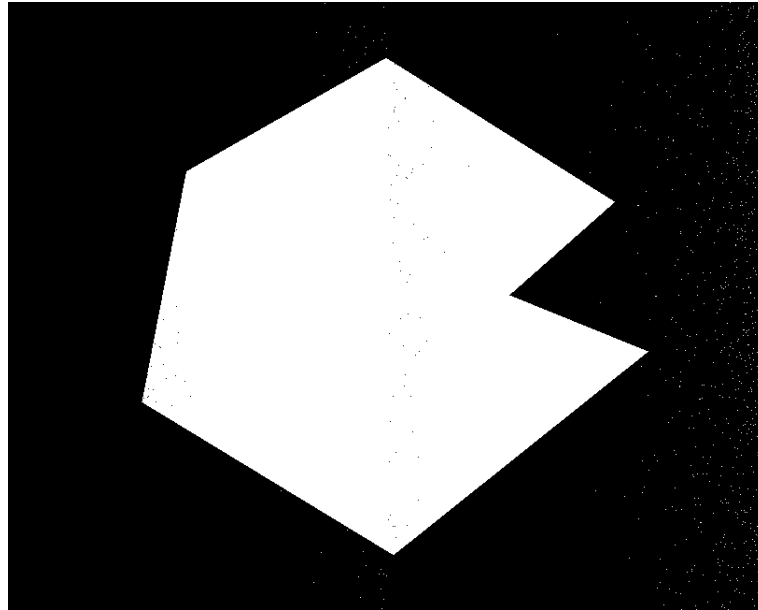
S1 = medfilt2(S, [3,3], 'symmetric');
figure; imshow(S1);
```

# Prostorno promenljivi prag

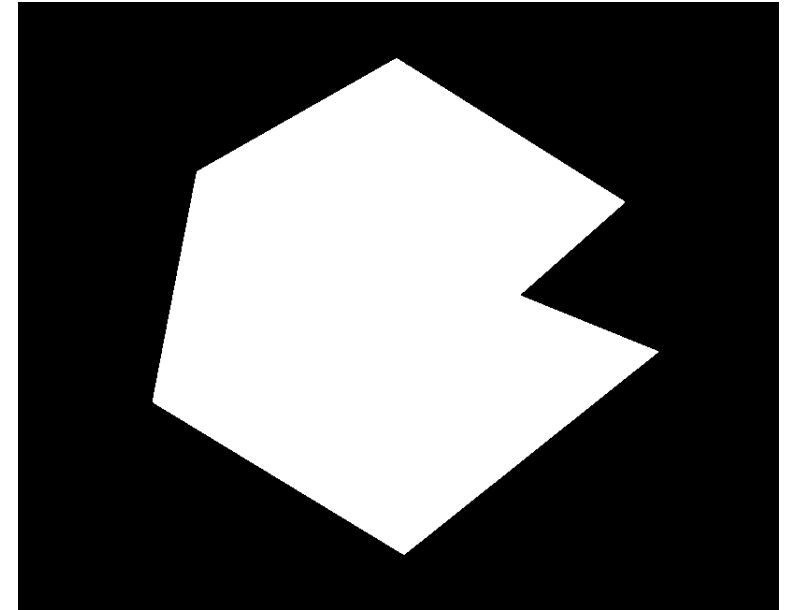
---



**globalni prag**



**lokalni prag**



**lokalni prag +  
medijan 3x3**

# Analiza slike

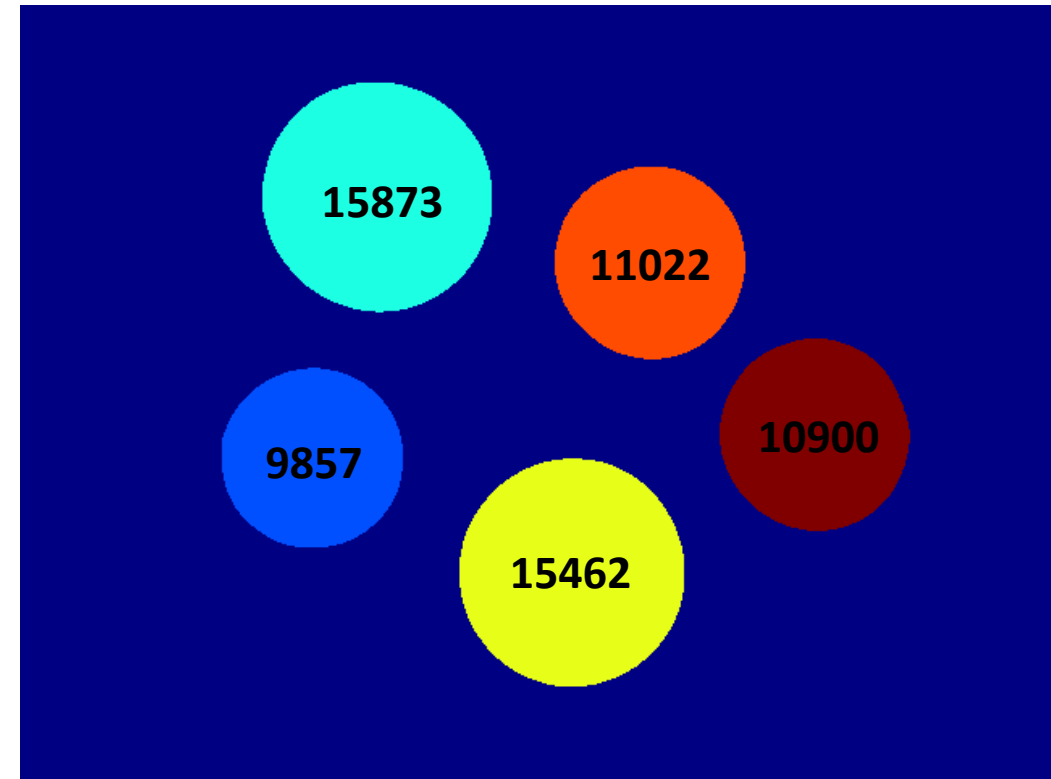
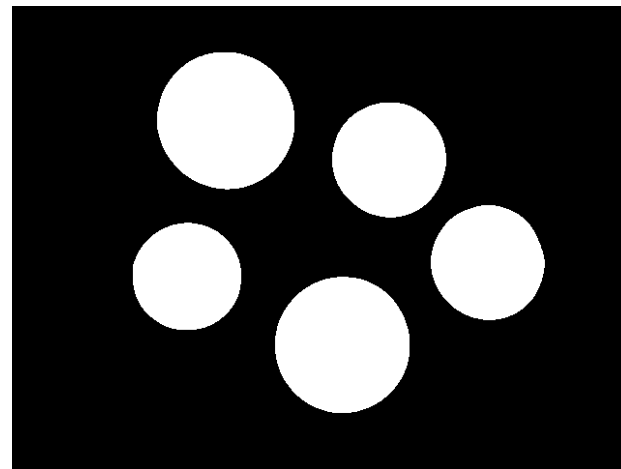
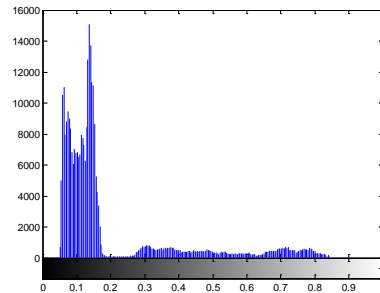
```
I = rgb2gray(imread('coins.jpg'));
I = im2double(I);
figure; imshow(I);

If = medfilt2(I, [13 13], 'symmetric');
figure; imshow(If);
figure; imhist(If); ylim('auto');

S = If > 0.2;
figure; imshow(S);
[coins, num] = bwlabel(S);
figure; imshow(coins, []); colormap('jet');

area = regionprops(S, 'Area');
```

# Analiza slike



# Analiza slike

```
I = rgb2gray(imread('coins.jpg'));
I = im2double(I); figure; imshow(I);
Ie = edge(I,'canny',0.4,4); figure; imshow(Ie);

[center, radius] = imfindcircles(Ie, [50, 150]);

coin2_pos = find(radius<60);
coin5_pos = find((radius>60)&(radius<70));
coin20_pos = find(radius>70);

figure; imshow(I);
viscircles(center(coin2_pos,:), radius(coin2_pos), 'Color', 'b', 'LineWidth', 2);
viscircles(center(coin5_pos,:), radius(coin5_pos), 'Color', 'g', 'LineWidth', 2);
viscircles(center(coin20_pos,:), radius(coin20_pos), 'Color', 'r', 'LineWidth', 2);
```

# Analiza slike

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