Saturday, April 17, 2021 10:05 PM

FURIJEOVA TRANSFORMACIJA KONTINUALNIH SIGNALA

FURIJEOVI REDOVI
- PERIODIČNI SIGNALI
- APERIODIČNI SIGNALI U KONAČNOM VREMENSKOM INTERVALU

} ISPUNJENI DIRIHLEOVI USLOVI

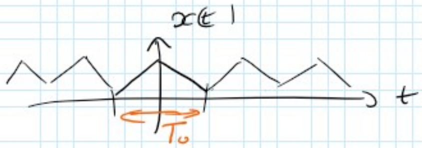
APERIODIČNI BESKONAČNOG TRAJANJA : FURIJOVA TRANSFORMACIJA

a) FURIJEOV INTEGRAL
b) POGLAVLJE 6.1

} FURIJEOV RED

$\Downarrow$

FURIJEOV INTEGRAL



$T_0 \to \infty$

$$\boxed{x(t) = \sum_{k=-\infty}^{+\infty} X[k] e^{jk\omega_0 t}}$$

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$

$x(t) \to$ KONTINUALNI SIGNAL

$$\mathcal{F}_t\{x(t)\} = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt = X(j\omega)$$

KRUŽNA UČESTANOST

$$\boxed{X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt}$$

$$x(t) \xleftrightarrow{\mathcal{F}_t} X(j\omega)$$

$$\boxed{x(t) = \left(\frac{1}{2\pi}\right) \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega}$$

$$\mathcal{F}_t^{-1}(X(j\omega)) = x(t)$$

$$X(j\omega) \xleftrightarrow{\mathcal{F}_t^{-1}} x(t)$$

$$X(j\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x(t) e^{j\omega t} dt$$

$$X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi f t} dt$$

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi f t} df$$

$X(j\omega) \leftrightarrow$
- GUSTINA FREKVENCIJSKOG SPEKTRA SIGNALA $x(t)$
- SPEKTAR SIGNALA $x(t)$ (FREKVENCIJSKI)

$$X(j\omega) = |X(j\omega)| e^{j\theta(\omega)}$$

$\theta(\omega)$ ← FAZNI SPEKTAR

$|X(j\omega)|$ ← AMPLITUDSKI SPEKTAR

AMPLITUDSKI SPEKTAR SIGNALA $x(t)$

△ $x(t) = \delta(t)$

$$X(j\omega) = \int_{-\infty}^{+\infty} \delta(t) e^{-j\omega t} dt = \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

$$\boxed{\mathcal{F}_t\{\delta(t)\} = 1}$$

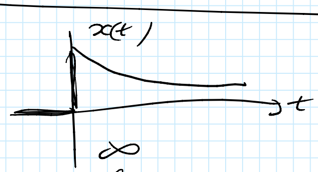
USLOVI EGZISTENCIJE FURIJEOVE TRANSF. (DIRIHLEOVI USLOVI)

① $\int_{-\infty}^{+\infty} |x(t)|\, dt < \infty$

② KONAČAN BROJ EKSTREMUMA U MA KOM KONAČNOM INTERVALU

③ KONAČAN BROJ PREKIDA I REDA U MA KOM KONAČNOM INTERVALU

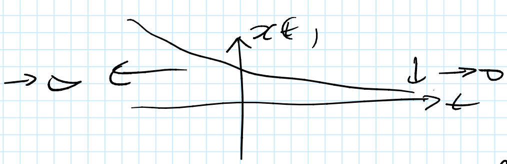
△ $x(t) = A \cdot e^{-at} \cdot u(t)$ $\boxed{a > 0}$



$$X(j\omega) = \int_0^{\infty} A e^{-at} e^{-j\omega t} dt = A \int_0^{\infty} e^{-(a+j\omega)t} dt$$

$$= \frac{-A}{a+j\omega} e^{-(a+j\omega)t} \Big|_0^{\infty} = \frac{A}{a+j\omega}$$

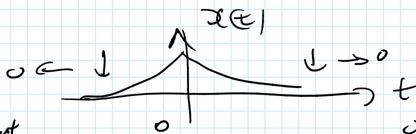
△ $\underline{x(t) = A \cdot e^{-at}}$ DA LI ~~ima~~ POSTOJI $\mathcal{F}_t\{x(t)\}$ NIJE ZADOVOLJEN ① DIRIH. USLOV



$$\int_{-\infty}^{+\infty} |x(t)|\, dt \to \infty$$

$$\lim_{\varepsilon \to \infty} \int_{-\varepsilon}^{\varepsilon} |x(t)|\, dt \to \infty$$

△ $x(t) = A e^{-a|t|} dt$ $a > 0$ $|t| = \begin{cases} t & \text{za } t > 0 \\ -t & \text{za } t < 0 \end{cases}$



$$X(j\omega) = \int_{-\infty}^{+\infty} A e^{-a|t|} e^{-j\omega t} dt = A \int_{-\infty}^{0} e^{-a(-t)} \cdot e^{-j\omega t} dt + A \int_0^{\infty} e^{-at} e^{-j\omega t} dt =$$

$$X(j\omega) = \int_{-\infty} A e^{-a|t|} e^{-j\omega t} dt = A \int_{-\infty}^{0} e^{-a(-t)} \cdot e^{-j\omega t} dt + \underbrace{A \int_{0} e^{-at} e^{-j\omega t} dt}_{\text{IZRAČUNALI PRETHODNO}} =$$

$$= A \int_{-\infty}^{0} e^{(a-j\omega)t} dt + \frac{1}{a+j\omega} =$$

$$= \boxed{\frac{A}{a-j\omega}} + \frac{A}{a+j\omega} = A \frac{2a}{a^2+\omega^2}$$

$a > 0$; a REALNO

$Re\{a\} > 0$; a KOMPLEKSNO

Δ $\quad x(t) = \begin{cases} e^{-at} & \text{za } t > 0 \\ -e^{at} & \text{za } t < 0 \end{cases}$ $\quad$ a REALNO, $a > 0$

$$X(j\omega) = -\frac{A}{a-j\omega} + \frac{A}{a+j\omega} = A \frac{-2j\omega}{a^2+\omega^2}$$

T** : AKO JE $x(t)$ REALNO — PARNO / NEPARNO

TADA JE $X(j\omega)$ — REALNO I PARNO / IMAGINARNO I NEPARNO

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt = \int_{-\infty}^{+\infty} x(t)(\cos\omega t - j\sin\omega t) dt$$

$$= \underbrace{\int_{-\infty}^{+\infty} x(t)\cos\omega t \, dt}_{J1} \underbrace{- j\int_{-\infty}^{+\infty} x(t)\sin\omega t \, dt}_{J2}$$

$x(t)$	$\cos\omega t$	$x(t)$	$\sin\omega t$
P	P	P	NP
P · P = P		P · NP = NP	
J1 ≠ 0, REALNO I PARNO		J2 = 0, NULA	
NP	P	NP	NP
NP · P = NP		NP · NP = P	
J1 = 0, NULA		J2 ≠ 0, IMAGINARAN NEPARAN	

TEOREME (OSOBINE)

$$X[k] = \frac{1}{T}\int_{\langle T\rangle} x(t) e^{-jk\omega_0 t}\,dt$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t}\,dt$$

1. LINEARNOST
2. $\mathcal{F}_t\{x(t-t_0)\} = e^{-j\omega t_0} X(j\omega)$
3. $X(j(\omega-\omega_0)) \xleftrightarrow{\mathcal{F}_t^{-1}} e^{j\omega_0 t} x(t)$

$$\mathcal{F}_t^{-1}\{X(j(\omega-\omega_0))\} = e^{j\omega_0 t} x(t)$$

$$\mathcal{F}_t\{e^{j\omega_0 t} x(t)\} = X(j(\omega-\omega_0))$$

4. $x(t)$: REALNO

$$|X(j\omega)| = |X(-j\omega)|$$

AMPLITUDSKI SPEKTAR JE PARNA F-JA OD ω

$$\theta(\omega) = -\theta(-\omega)$$

(DOKAZATI TEOREMU ZA PARNOST I NEPARNOST OD $x(t)$)

Δ POVRŠINA ISPOD FUNKCIJE

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t}\,dt \quad \Big/ \lim_{\omega\to 0}$$

$$X(0) = \int_{-\infty}^{+\infty} x(t)\,dt$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t}\,d\omega = x(t) \quad \Big/ \lim_{t\to 0}$$

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\,d\omega = x(0)$$

$$\int_{-\infty}^{+\infty} X(j\omega)\,d\omega = \underline{2\pi x(0)}$$

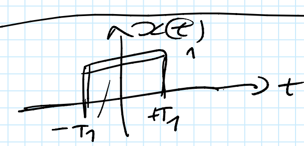
PRIMER:

$$f(x) = 10 \operatorname{sinc}\frac{x+4}{7} \qquad \int_{-\infty}^{+\infty} f(x)\,dx$$

$$I = \int_{-\infty}^{+\infty} 10 \operatorname{sinc}\frac{x+4}{7}\,dx = 10\int_{-\infty}^{+\infty} \operatorname{sinc}\frac{t}{7}\,dx$$

UZETI IZ TABLICE

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases}$$



$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t}\,dt = \int^{T_1} 1\cdot e^{-j\omega t}\,dt =$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt = \int_{-T_1}^{T_1} 1 \cdot e^{-j\omega t} dt =$$

$$= -\frac{1}{j\omega} e^{-j\omega t} \Big|_{-T_1}^{T_1} = \frac{e^{j\omega T_1} - e^{-j\omega T_1}}{j\omega} = 2 \frac{\sin \omega T_1}{\omega} =$$

$$= 2T_1 \frac{\sin \pi \cdot \left(\frac{\omega T_1}{\pi}\right)}{\pi \left(\frac{\omega T_1}{\pi}\right)} = \boxed{2T_1 \operatorname{sinc}\left(\frac{\omega T_1}{\pi}\right)}$$

$\operatorname{sinc} \frac{x}{7}$ $\quad dx \quad , \quad d\omega \Rightarrow x = \omega$

$$\frac{\omega T_1}{\pi} = \frac{x}{7} \qquad \boxed{\frac{T_1}{\pi} = \frac{1}{7}}$$

$$\frac{1}{\cancel{2}\pi} \int_{-\infty}^{+\infty} \left(\cancel{2}T_1 \operatorname{sinc}\left(\frac{\omega T_1}{\pi}\right)\right) e^{j\omega t} d\omega = \operatorname{rect}\left(\frac{t}{2T_1}\right)$$

$$\frac{1}{\pi} \int_{-\infty}^{+\infty} T_1 \operatorname{sinc} \frac{\omega T_1}{\pi} d\omega = \operatorname{rect}\left(\frac{0}{2T_1}\right) = 1$$

$$\frac{T_1}{\pi} \int_{-\infty}^{+\infty} \operatorname{sinc} \frac{\omega T_1}{\pi} d\omega = 1$$

$$\frac{1}{7} \int_{-\infty}^{+\infty} \operatorname{sinc} \frac{x}{7} dx = 1 \qquad \boxed{\int_{-\infty}^{+\infty} \operatorname{sinc} \frac{x}{7} dx = 7}$$

$$I = 10 \cdot 7 = 70$$

▽ POVRŠINA ISPOD FUNKCIJE

GENERALIZOVANA FURIJEOVA TRANSFORMACIJA

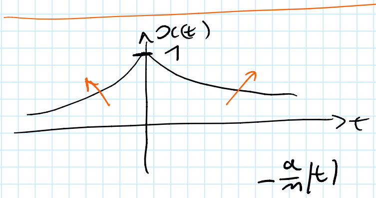
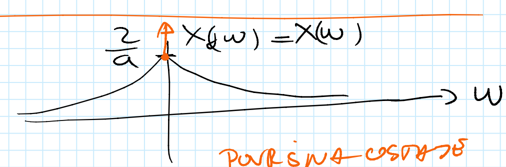
$$\boxed{\begin{gathered} \delta(t) \\ \mathcal{F}_t\{\delta(t)\} = 1 \end{gathered}}$$

$x(t) = e^{-a|t|}$ $\quad a > 0$, REALNO

$$X(j\omega) = \frac{2a}{a^2 + \omega^2}$$

$$\int_{-\infty}^{+\infty} x(t) dt = X(0) = \frac{2}{a}$$

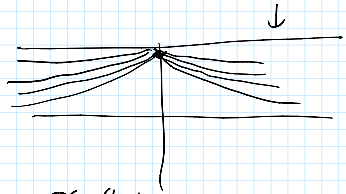
$$\int_{-\infty}^{+\infty} X(j\omega) d\omega = 2\pi \, x(0) = 2\pi$$

$x(t)$	$X(j\omega)$
	

POVRŠINA OSTAJE ISTA $= 2\pi$

SMANJUJEMO a) $\quad x_n(t) = e^{-\frac{a}{n}|t|}$

$n \to \infty$



$\lim_{n\to\infty} x_n(t) = 1$

POVRŠINA OSTAJE ISTA $= 2\pi$

$a \to \infty$

$X(j\omega) \to 2\pi \cdot \delta(\omega)$

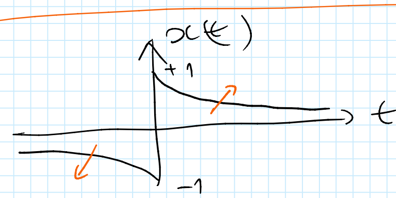
$$\mathcal{F}_t\{1\} = 2\pi\,\delta(\omega)$$

$$\frac{1}{\cancel{2\pi}}\int_{-\infty}^{+\infty} \cancel{2\pi}\,\delta(\omega)e^{j\omega t}\,d\omega = \int_{-\infty}^{+\infty}\delta(\omega)\,d\omega = 1$$

$$\boxed{\begin{array}{ll} \mathcal{F}_t\{\delta(t)\} = 1 & \mathcal{F}_t^{-1}\{\delta(\omega)\} = \frac{1}{2\pi} \\ \mathcal{F}_t\{1\} = 2\pi\,\delta(\omega) & \end{array}}$$

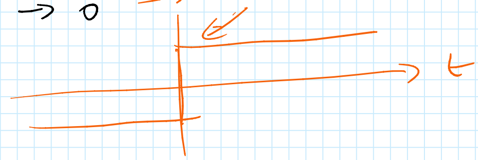
$$x(t) = \begin{cases} e^{-at} & t > 0 \\ -e^{at} & t < 0 \end{cases}$$

$$X(j\omega) = -\frac{2j\omega}{a^2+\omega^2}$$



$$\mathcal{F}_t\{\mathrm{sign}(t)\} = -\frac{2j}{\omega} = \frac{2}{j\omega}$$

$a \to 0 \quad \to x(t) \to \mathrm{sign}(t)$



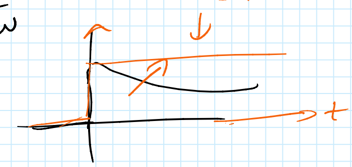
$$u(t) = \frac{1+\mathrm{sign}(t)}{2}$$

$$\mathcal{F}_t\{u(t)\} = \mathcal{F}_t\left\{\frac{1+\mathrm{sign}(t)}{2}\right\}$$

$$= \mathcal{F}_t\{\tfrac{1}{2}\} + \mathcal{F}_t\{\tfrac{\mathrm{sign}(t)}{2}\} =$$

$$= \pi\,\delta(\omega) + \frac{1}{j\omega}$$

$$\boxed{\mathcal{F}_t\{u(t)\} = \pi\,\delta(\omega) + \frac{1}{j\omega}}$$



TEOREMA O POMERANJU U ω DOMENU

$$\mathcal{F}_t\{e^{j\omega_0 t}x(t)\} = X(j(\omega-\omega_0))$$

$x(t) = 1 \qquad \mathcal{F}_t\{x(t)\} = 2\pi\,\delta(\omega)$

$x(t)=1 \quad \mathcal{F}_t\{x(t)\}=2\pi\delta(\omega)$

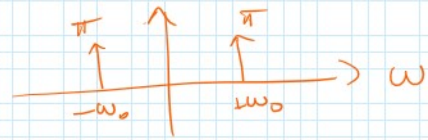
$\mathcal{F}_t\{e^{j\omega_0 t}\}=2\pi\delta(\omega-\omega_0)$

$\mathcal{F}_t\{\cos\omega_0 t+j\sin\omega_0 t\}=2\pi\delta(\omega-\omega_0)$

$\mathcal{F}_t\{e^{-j\omega_0 t}\}=2\pi\delta(\omega+\omega_0)$

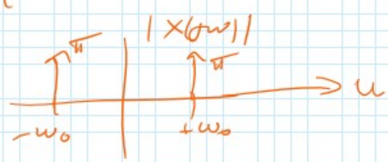
$\mathcal{F}_t\left\{\frac{e^{j\omega_0 t}+e^{-j\omega_0 t}}{2}\right\}=\mathcal{F}_t\{\cos\omega_0 t\}=\frac{2\pi}{2}\left(\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\right)$

$\mathcal{F}_t\{\cos\omega_0 t\} \longrightarrow$



$\mathcal{F}_t\left\{\underbrace{\frac{e^{j\omega_0 t}-e^{-j\omega_0 t}}{2j}}_{X(j\omega)}\right\}=-j\pi\{\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\}$

$=j\pi\{\delta(\omega+\omega_0)-\delta(\omega-\omega_0)\}$



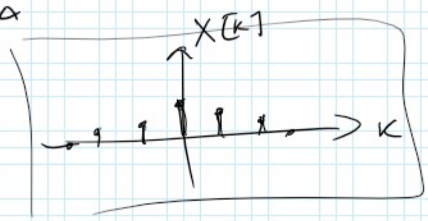
$\boxed{x(t)e^{-a|t|} \quad a\to 0}$

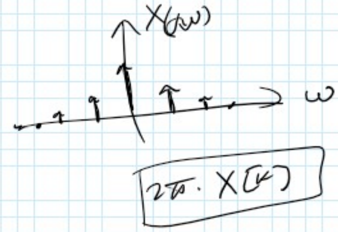
$x(t) \longrightarrow$ PERIODIČAN

$x(t)=\sum_{k=-\infty}^{+\infty}X[k]e^{+jk\omega_0 t}$

FURIJEOVA TRANSFORMACIJA PERIODIČNOG SIGNALA

$\mathcal{F}_t\{x(t)\}=X(j\omega)=2\pi\sum_{k=-\infty}^{+\infty}X[k]\delta(\omega-k\omega_0)$





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(5) SKALIRANJE VREMENSKE OSE

$\mathcal{F}_t\{x(t)\}=X(j\omega)$

$\mathcal{F}_t\{\ \} = \frac{1}{\ }X\left(j\frac{\omega}{\ }\right)$

KOMPESIJA U JEDNOM DOMENU ODGOVARA EKSPANZIJI U

OSE

$$\mathcal{F}_t\{x(t)\} = X(j\omega)$$

$$\mathcal{F}_t\{x(at)\} = \frac{1}{|a|} X\left(j\frac{\omega}{a}\right)$$

... ODGOVARA EKSPANZIJI U TRANSFORMACIONOM" DOMENU

VAŽI I OBRNUTO

12 : 32

(6) SKALIRANJE ω OSE

$$\mathcal{F}_t^{-1}\{X(j\omega)\} = x(t)$$

$$\mathcal{F}_t^{-1}\{X(ja\omega)\} = \frac{1}{|a|} x\left(\frac{t}{a}\right)$$

(7) FURIJEOVA TRANSF KONJ. KOMPLEKSNOG SIGN.

$$\mathcal{F}_t\{x^*(t)\} = X^*(-j\omega)$$

(8) FURIJEOVA TRANSFOMACIJA KONVOLUCIJE 2 SIGNALA

$$\mathcal{F}_t\{x(t)\} = X(j\omega) \quad \mathcal{F}_t\{h(t)\} = H(j\omega)$$

$$\mathcal{F}_t\{y(t)\} = \mathcal{F}_t\{x(t) * h(t)\} = X(j\omega) \cdot H(j\omega)$$

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

$$Y(j\omega) = \int_{-\infty}^{+\infty} (x(t) * h(t)) e^{-j\omega t} dt = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x(\tau) h(t-\tau) d\tau \cdot e^{-j\omega t} dt$$

$$= \int_{-\infty}^{+\infty} x(\tau) \underbrace{\left(\int_{-\infty}^{+\infty} h(t-\tau) e^{-j\omega t} dt\right)}_{e^{-j\omega\tau} H(j\omega)} d\tau = \int_{-\infty}^{+\infty} x(\tau) e^{-j\omega\tau} H(j\omega) d\tau$$

$$= H(j\omega) \underbrace{\int_{-\infty}^{+\infty} x(\tau) e^{-j\omega\tau} d\tau}_{X(j\omega)} = H(j\omega) \cdot X(j\omega)$$

$$\boxed{y(t) = x(t) * h(t)}$$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$Y(j\omega) = X(j\omega) H(j\omega)$$

$$\boxed{y(t) = \mathcal{F}_t^{-1}\{X(j\omega) \cdot H(j\omega)\}}$$

(9) FURIJEOVA TRANSFORMACIJA

(9) FURIJEOVA TRANSFORMACIJA PROIZVODA DVA SIGNALA

$$Y(j\omega) = X(j\omega) * H(j\omega) \qquad y(t)$$

$$y(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} (X(j\omega) * H(j\omega)) e^{j\omega t} dt =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty}\left(\int_{-\infty}^{+\infty} X(j\theta) H(j(\omega-\theta)) d\theta\right) e^{j\omega t} d\omega =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\theta) \underbrace{\left(\int_{-\infty}^{+\infty} H(j(\omega-\theta)) e^{j\omega t} d\omega\right)} d\theta =$$

$$= \frac{1}{\cancel{2\pi}}\int_{-\infty}^{+\infty} X(j\theta)\left(\cancel{2\pi} e^{j\theta t} h(t)\right) d\theta = h(t) \underbrace{\int_{-\infty} X(j\theta) e^{j\theta t} d\theta}_{2\pi x(t)}$$

$$= 2\pi \, x(t) \cdot h(t)$$

$$\mathcal{F}_t\{x(t) \cdot h(t)\} = \frac{1}{2\pi} X(j\omega) * H(j\omega)$$

$$x(t) = \cos\omega_0 t \cdot u(t)$$

$$\mathcal{F}_t\{\cos\omega_0 t\} = \pi(\delta(\omega-\omega_0) + \delta(\omega+\omega_0))$$

$$\mathcal{F}_t\{u(t)\} = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$\mathcal{F}_t\{\cos\omega_0 t \cdot u(t)\} = \boxed{\frac{1}{2\pi}\mathcal{F}_t\{\cos\omega_0 t\} * \mathcal{F}_t\{u(t)\}}$$

$$X(j\omega) = \frac{1}{2\cancel{\pi}}\left(\pi\delta(\omega) + \frac{1}{j\omega}\right) * \left(\cancel{\pi}(\delta(\omega-\omega_0) + \delta(\omega+\omega_0))\right)$$

$$= \frac{\pi}{2}\delta(\omega) * (\delta(\omega-\omega_0) + \delta(\omega+\omega_0)) + \frac{1}{2}\frac{1}{j\omega} * (\delta(\omega-\omega_0) + \delta(\omega+\omega_0))$$

$$= \frac{\pi}{2}(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)) + \frac{1}{2}\left(\frac{1}{j(\omega-\omega_0)} + \frac{1}{j(\omega+\omega_0)}\right) =$$

$$= \frac{-j\omega}{\omega^2 - \omega_0^2} + \frac{\pi}{2}(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)) \qquad \boxed{x(t) * \delta(t-t_0) = x(t-t_0)}$$

$$= \frac{-j\omega}{\omega^2 - \omega_0^2} + \frac{\pi}{2}\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)$$

$$= \frac{j\omega}{\omega_0^2 - \omega^2} + \frac{\pi}{2}\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)$$

$$= \frac{j\omega}{\omega_0^2 + (j\omega)^2} + \frac{\pi}{2}\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)$$

DIGRESIJA

$$x(t) * \delta(t-t_0) = x(t-t_0)$$

$$\delta(t) \to \boxed{h(t)} \to h(t)$$

$$\delta(t-t_0) \to \boxed{h(t)} \to h(t-t_0)$$

$$\delta(t-t_0) \to \boxed{x(t)} \to x(t-t_0)$$

$$x(t) * \delta(t-t_0) = x(t-t_0)$$

$X(j\omega)$ $\quad$ $j\omega = s$

$$\boxed{X(s) = \frac{s}{\omega_0^2 + s^2} + \frac{\pi}{2}\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)}$$

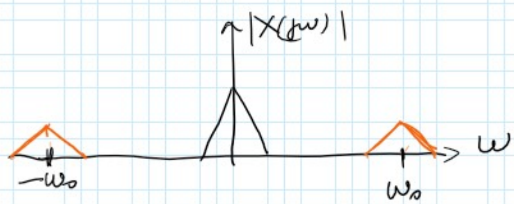
$$\mathcal{F}_t\{\sin\omega_0 t \cdot u(t)\} = \frac{\omega_0}{\omega_0^2 + s^2} + \frac{\pi}{2j}\left(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right)$$

$$\mathcal{F}_t\{x(t) \cdot \cos\omega_0 t\} = \frac{1}{2\pi} X(j\omega) * \mathcal{F}_t\{\cos\omega_0 t\} =$$

$$= \frac{1}{2} X(j\omega) * \left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right) =$$

$$= \frac{1}{2}\left(X(j(\omega-\omega_0)) + X(j(\omega+\omega_0))\right)$$

$|X j\omega|$



(10) FURIJOVA TRANSF. IZVODA

$$\mathcal{F}_t\left\{\frac{d}{dt}x(t)\right\} = \mathcal{F}_t\{D\,x(t)\} = j\omega \cdot X(j\omega)$$

$$\mathcal{F}_t\{D^k x(t)\} = (j\omega)^k X(j\omega)$$

$h(t) \to$ IMPULSNI ODZIV $\quad \to \boxed{h(t)} \to$

$$\boxed{\int_{-\infty}^{+\infty} |h(t)|\,dt < \infty}$$

$H(j\omega) \to$ FREKVENCIJSKI ODZIV SISTEMA

$$\boxed{H(j\omega) = \mathcal{F}_t\{h(t)\}}$$

$s = j\omega$

PRENOSNA... ISKI ODZIV SISTEMA $\quad H(j\omega) = \mathcal{F}_t\{h(t)\}$

$s = j\omega$

PRENOSNA FUNKCIJA $\quad \boxed{P(D)\,y(t) = Q(D)\,x(t)} \; / \mathcal{F}_t$

$$P(s)\,Y(s) = Q(s)\cdot X(s)$$

$$Y(s) = \frac{Q(s)}{P(s)}\cdot X(s) = H(s)\cdot X(s)$$

$$H(s) = \frac{Q(s)}{P(s)} = \mathcal{F}_t\{h(t)\}$$

$$H(D) \rightleftarrows H(s)$$

$$y(t) = h(t) * x(t) \; / \; \mathcal{F}_t$$

$$Y(s) = H(s)\cdot X(s)$$

$$\boxed{h(t) = \mathcal{F}_t^{-1}\left\{\frac{Q(s)}{P(s)}\right\}}$$

$$y(t) = \mathcal{F}_t^{-1}\left\{\frac{Q(s)}{P(s)}\right\}$$

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11 FURIJEOVA TRANSFORMACIJA INTEGRALA FUNKCIJE

$$\boxed{x(t) * u(t)} = \int_{-\infty}^{+\infty} x(\tau)\,u(t-\tau)\,d\tau = \boxed{\int_{-\infty}^{t} x(\tau)\,d\tau}$$

$$\mathcal{F}_t\{x(t) * u(t)\} = X(j\omega)\cdot U(j\omega) =$$

$$= X(j\omega)\cdot\left(\frac{1}{j\omega} + \pi\delta(\omega)\right) = \boxed{\frac{X(j\omega)}{j\omega} + \pi X(0)\,\delta(\omega)}$$

$$\mathcal{F}_t\left\{\int_{-\infty}^{t} x(\tau)\,d\tau\right\} = \frac{X(j\omega)}{j\omega} + \pi X(0)\,\delta(\omega)$$

1:30

12 IZVOD U KOMPLEKSNOM DOMENU

$$\frac{d}{d\omega}X(j\omega) = \frac{d}{d\omega}\int_{-\infty}^{+\infty} x(t)\,e^{-j\omega t}\,dt = \int_{-\infty}^{+\infty} x(t)\,\frac{d}{d\omega}e^{-j\omega t}\,dt =$$

$$\frac{d}{d\omega} X(j\omega) = \frac{d}{d\omega} \int_{-\infty} x(t) e^{-j\omega t} dt = \int_{-\infty} x(t) \frac{d}{d\omega} e^{-j\omega t} dt =$$

$$= -j \int_{-\infty}^{+\infty} t\, x(t) e^{-j\omega t} dt = -j \mathcal{F}_t \{ t\, x(t) \}$$

$$\mathcal{F}_t \{ t\, x(t) \} = +j \frac{d}{d\omega} X(j\omega)$$

$$\mathcal{F}_t \{ t^n x(t) \} = j^n \frac{d^n}{d\omega^n} X(j\omega)$$

$a > 0$, REALNO

$$\boxed{\mathcal{F}_t \left\{ \frac{t^{n-1}}{(n-1)!} e^{-at} u(t) \right\} = \frac{1}{(a+j\omega)^n}}$$

13 PASERVALOVA TEOREMA

SPEKTRALNA GUSTINA ENERGIJE

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega = \int_{-\infty}^{+\infty} |X(f)|^2 df$$