PERIODIČNI SIGNALI $T_0 = T_F$

$\mathcal{F}_s\{x(t)\} = \underline{X[k]}$ $y(t) = O\{x(t)\}$

$Y[k] = \mathcal{F}_s\{y(t)\}$

7 $y(t) = D\,x(t) =$

$= \frac{d}{dt} x(t)$

OSNOVNI PERIOD OD $x(t)$ i $y(t)$ $\underline{T_0}$

$$x(t) = \sum_{k=-\infty}^{+\infty} X[k] e^{jk\omega_0 t}$$

$$\underbrace{D\,x(t)}_{y(t)} = D\left(\sum_{k=-\infty}^{+\infty} X[k]\, e^{jk\omega_0 t}\right) =$$

$$= \sum_{k=-\infty}^{+\infty} X[k]\, D\left(e^{jk\omega_0 t}\right) =$$

$$= \sum_{k=-\infty}^{+\infty} \underbrace{\left(X[k]\cdot(jk\omega_0)\right)}_{Y[k]} e^{jk\omega_0 t} =$$

$$= \sum_{k=-\infty}^{+\infty} Y[k]\, e^{jk\omega_0 t}$$

$$\boxed{Y[k] = jk\omega_0 X[k]}$$

TEOREMA O DIFERENCIRANJU

$$\mathcal{F}_s\{D\,x(t)\} = \mathcal{F}_s\{y(t)\} = jk\omega_0 X[k]$$

$$\mathcal{F}_s\{D^n x(t)\} = (jk\omega_0)^n X[k]$$

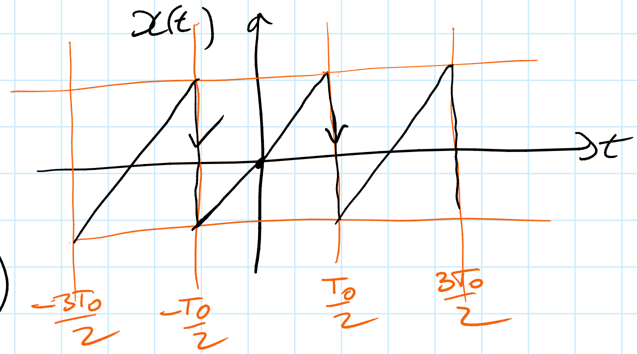
- $x(t)$ GLATKA FUNKCIJA (DIFERENCIJABILNA $\underline{n}$ PUTA)
- $x(t)$ IMA PREKIDE

POTREBNO JE GENERALIZOVANO DIFERENCIRANJE

$x_F(t) = t \qquad -\frac{T_0}{2} < t < \frac{T_0}{2}$ OSNOVNA PERIODA $x(t)$ JE T_0

$\frac{d}{}\, x_F(t) = 1 \qquad x(t)$

$$\frac{d}{dt}x_F(t) = 1$$



$$x_F(t) =$$
$$= t\left(u\left(t+\frac{T_0}{2}\right) - u\left(t-\frac{T_0}{2}\right)\right)$$

$$Dx_F(t) = 1\cdot\left(u\left(t+\frac{T_0}{2}\right) - u\left(t-\frac{T_0}{2}\right)\right) + t\left(\delta\left(t+\frac{T_0}{2}\right) - \delta\left(t-\frac{T_0}{2}\right)\right) = u\left(t+\frac{T_0}{2}\right) - u\left(t-\frac{T_0}{2}\right) + \left(-\frac{T_0}{2}\delta\left(t+\frac{T_0}{2}\right) - \frac{T_0}{2}\delta\left(t-\frac{T_0}{2}\right)\right)$$

8)

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau \quad \downarrow \text{PERIODIČNO}$$
$$\mathcal{F}_S\{x(t)\} = X[k]$$
$$\Big\} \quad Y[k] = ?$$

$x(t), y(t)$ — ISTA PERIODA T_0

$$\boxed{\int_{-\infty}^{t} x(\tau)\,d\tau} \leftarrow$$ POTREBNO JE DA POSTOJI

① $X[0] = 0$ ILI INTEGRAL NE KONVERGIRA ILI SE INTEGRACIJOM NE DOBIJA PERIODIČNA FUNKCIJA

② $X[0] \neq 0$

$$\int_{-\infty}^{t} (\ )\,d\tau = D^{-1}$$

$$D \rightarrow jk\omega_0$$
$$D^{-1} \rightarrow \frac{1}{jk\omega_0}$$

$$\mathcal{F}_S\left\{\int_{-\infty}^{t} x(\tau)\,d\tau\right\} = \frac{1}{jk\omega_0}X[k] = Y[k]$$

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau = \int_{-\infty}^{t}\left(\sum_{k=-\infty}^{+\infty} X[k]e^{jk\omega_0\tau}\right)d\tau =$$

$$\sum^{+\infty} X[k] \int^{t} e^{jk\omega_0\tau}\,d\tau = \sum^{+\infty} X[k]\frac{1}{jk\omega_0}e^{jk\omega_0\tau}\Big|^{t} =$$

$$= \sum_{k=-\infty}^{+\infty} X[k] \int_{-\infty}^{t} e^{jk\omega_0 \tau} d\tau = \sum_{k=-\infty}^{+\infty} X[k] \frac{1}{jk\omega_0} e^{jk\omega_0 t} \Big|_{-\infty} =$$

$$= \sum_{k=-\infty}^{+\infty} X[k] \cdot \frac{1}{jk\omega_0} e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} Y[k] e^{jk\omega_0 t}$$

$$\boxed{Y[k] = \frac{1}{jk\omega_0} X[k]}$$

"INTEGRACIJA $x(t)$ PO VREMENU"

(9) RAZVOJ U RED PROIZVODA DVA SIGNALA

$$z(t) = x(t) \cdot y(t)$$

$$\underbrace{T_0 \quad T_{0x} = T_{0y} = T_0}_{\text{OSNOVNE PERIODE}}$$

$$\boxed{\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}}$$

$$z(t) = \sum_{k=-\infty}^{+\infty} Z[k] e^{jk\omega_0 t}$$

$$Z[k] = \frac{1}{T_0} \int_{T_0} z(t) e^{-jk\omega_0 t} dt = \frac{1}{T_0} \int_{T_0} x(t) \cdot y(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{T_0} \int_{T_0} x(t) \underbrace{\left[\sum_{\ell=-\infty}^{+\infty} Y[\ell] e^{j\ell\omega_0 t} \right]}_{y(t)} e^{-jk\omega_0 t} dt =$$

$$= \sum_{\ell=-\infty}^{+\infty} Y[\ell] \underbrace{\frac{1}{T_0} \int_{T_0} x(t) e^{-j\overbrace{(k-\ell)}^{m}\omega_0 t} dt}_{X[m] = X[k-\ell]} = \sum_{\ell=-\infty}^{+\infty} Y[\ell] X[k-\ell]$$

KONVOLUCIJA!

$$= Y[k] * X[k]$$

$y(t), x(t)$ PERIODIČNI SA T_0

$Y[k], X[k]$

$$z(t) = y(t) \cdot x(t) \xleftrightarrow{F_S} Z[k] = Y[k] * X[k]$$

$$z(t) = y(t), x(t) \xleftrightarrow{F_s} Z[k] = Y[k] * X[k]$$

KADA OSNOVNE PERIODE $x(t)$ i $y(t)$ NISU JEDNAKE (KADA $T_{0x} \neq T_{0y}$) TADA JE POTREBNO NAĆI ZAJEDNIČKU PERIODU $\underline{T_0}$

(PROMENA PERIODE IZRAČUNAVANJA)

(10) RAZVOJ U F RED „KONVOLUCIJE" DVA SIGNALA

$x(t), y(t)$ PERIODA $T_{0x} = T_{0y} = T_0$

$$\boxed{x(t) \circledast y(t) = \int_{T_0} x(\tau) y(t-\tau) d\tau}$$

$$z(t) = x(t) \circledast y(t) \qquad \boxed{T_{0x} = T_{0y}} = T_{0z} = T_0$$

$$z(t) \to Z[k] \qquad x(t) \to X[k] \qquad y(t) \to Y[k]$$

$Z[k]$?

$$F_s\{\overbrace{x(t) \circledast y(t)}^{z(t)}\} = T_0 X[k] Y[k] = Z[k]$$

IZRAZ TEOREME

$$z(t) = \sum_{k=-\infty}^{+\infty} Z[k] e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} T_0 X[k] Y[k] e^{jk\omega_0 t} =$$

$$= T_0 \sum_{k=-\infty}^{+\infty} X[k] Y[k] e^{jk\omega_0 t} = \cancel{T_0} \sum_{k=-\infty}^{+\infty} \overbrace{\left(\frac{1}{\cancel{T_0}} \int_{T_0} x(\tau) e^{-jk\omega_0 \tau} d\tau \right)}^{X[k]} Y[k] e^{jk\omega_0 t}$$

$$= \int_{T_0} x(\tau) \underbrace{\left[\sum_{k=-\infty}^{+\infty} Y[k] e^{-jk\omega_0 (t-\tau)} \right]}_{y(t-\tau)} d\tau = \underbrace{\int_{T_0} x(\tau) y(t-\tau) d\tau}_{x(t) \circledast y(t)}$$

$$F_s\{x(t) \underset{T_0}{\circledast} y(t)\} = T_0 X[k] \cdot Y[k]$$

$x_F(t) \to$ APERIODIČAN SIGNAL $\qquad$ $x(t), y(t)$ PERIODIČNI SA T_0

SAMO JEDNA PERIODA $\underline{x(t)}$ VAN TE PERIODE JE $\emptyset$

$$\int_{-\infty}^{+\infty} x_F(\tau) y(t-\tau) d\tau = \int x(\tau) y(t-\tau) d\tau$$

SAMO JEDNA PERIODA $x(t)$ VAN JE PERIODE JE 0

$$\underbrace{\int_{-\infty}^{+\infty} x_F(\tau)\, y(t-\tau)\, d\tau}_{x_F(t) * y(t)} = \int_{T_0} x(\tau)\, y(t-\tau)\, d\tau$$

$$x_F(t) * y(t) = x(t) \circledast y(t)$$

---

(11) FURIJEOV RAZVOJ KONJUGOVANO KOMPLEKSNOG SIGNALA

$$\mathcal{F}_S\{x(t)\} = X[k] \Rightarrow \mathcal{F}_S\{x^*(t)\} = X^*[-k]$$

$$x^*(t) = (x(t))^* = \left(\sum_{k=-\infty}^{+\infty} X[k] e^{jk\omega_0 t}\right)^* = \qquad (A+C+D)^* = A^*+C^*+D^*$$

$$= \sum_{k=-\infty}^{+\infty} X^*[k] e^{-jk\omega_0 t} \qquad m = -k$$

$$= \sum_{m=+\infty}^{-\infty} X^*[-m] e^{jm\omega_0 t} = \underbrace{\sum_{m=-\infty}^{+\infty} X^*[-m] e^{jm\omega_0 t}}_{\text{DEFINICIJA RAZVOJA U RED}}$$

$$\mathcal{F}_S\{x^*(t)\} = X^*[-m] = X^*[-k]$$

$$\mathcal{F}_S\{x^*(t)\} = X^*[-k]$$

(12) PASERVALOVA TEOREMA

$x(t) \rightarrow$ PERIODIČNO SA T_0

$$P_{SR} = \frac{1}{T_0}\int_{T_0} |x(t)|^2\, dt \qquad \boxed{z \cdot z^* = |z|^2}$$

$$\mathcal{F}_S\{x^*(t)\} = X^*[-k]$$

$$P_{SR} = \frac{1}{T_0}\int_{T_0} \underbrace{x(t) \cdot x^*(t)}\, dt$$

$$\frac{1}{T_0}\int_{T_0} x(t) \cdot x^*(t) e^{-jk\omega_0 t}\, dt = \sum_{q=-\infty}^{+\infty} X[q] \cdot X^*[-(k-q)] =$$

$$\sum^{+\infty} X[q]\, X^*[q-k] \qquad \Big| \qquad k = 0$$

T_0

$$= \sum_{\ell=-\infty}^{+\infty} X[\ell] X^*[\ell-k] \Big|_{k=0}$$

$X^*[\ell]$

$$\frac{1}{T_0}\int_{T_0} x(t)x^*(t)\,dt = \sum_{k=-\infty}^{+\infty} X[k]X^*[k] = \sum_{k=-\infty}^{+\infty} |X[k]|^2$$

$$P_{sr} = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |X[k]|^2$$

SREDNJA SNAGA JEDNAKA JE SUMI KVADRATA AMPLITUDA HARMONIKA

$$x_{eff} = \sqrt{\frac{1}{T_0}\int_{T_0} |x(t)|^2 dt} = \sqrt{\sum_{k=-\infty}^{+\infty} |X[k]|^2}$$

EFEKTIVNA ILI SREDNJA KVADRATNA VREDNOST ~~REALNOG~~ SIGNALA

REALNI SIGNALI:

$x(t)$ REALNO, PERIODIČNO

$$|X[k]| = |X[-k]|$$

$$\boxed{X[0] = \frac{1}{T_0}\int_{T_0} x(t)\,dt}$$

$$P_{sr} = X[0]^2 + 2\sum_{k=1}^{\infty} |X[k]|^2$$

$$\boxed{c[k] = \sqrt{A^2[k] + B^2[k]} = 2|X[k]|}$$

$$\varphi_k = -\operatorname{arctg}\left(\frac{B[k]}{A[k]}\right) = \arg(X[k])$$

$$X[k] = \frac{A[k] - jB[k]}{2} \qquad k > 0$$

$$X[0] = A[0] = C[0]$$

$$P_{sr} = C[0]^2 + \sum_{k=1}^{\infty} \frac{|2X[k]|^2}{2} =$$

$$= C[0]^2 + \sum_{k=1}^{\infty} \frac{(C[k])^2}{2}$$

SNAGA JEDNOSMERNE KOMPONENTE

SUMA KVADRATA AMPLITUDA HARMONIKA PODELJENIH SA DVA

KVADRAT EFEKTIVNE VREDNOSTI SIGNALA $x(t) \in \mathbb{R}$

$$P_{SR} = C[0]^2 + \sum_{k=1}^{\infty} \left(\frac{C[k]}{\sqrt{2}}\right)^2$$

$$\boxed{\frac{C[k]}{\sqrt{2}}}$$ EFEKTIVNA VREDNOST K-TOG HARMONIKA

SUMA KVADRATA

SUMA KVADRATA EFEKTIVNIH VREDNOSTI HARMONIKA

K-TOG HARMONIKA

$$P_{se} = A[0]^2 + \sum_{k=1}^{\infty} \frac{\left(A[k]^2 + B[k]^2\right)}{2}$$

GIBSOV FENOMEN

$$x_N(t) = \sum_{k=-N}^{N} X[k] e^{jk\omega_0 t}$$

$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$

$x(t) \longrightarrow T_0$

$$F_s\{x(t)\} = X[k]$$

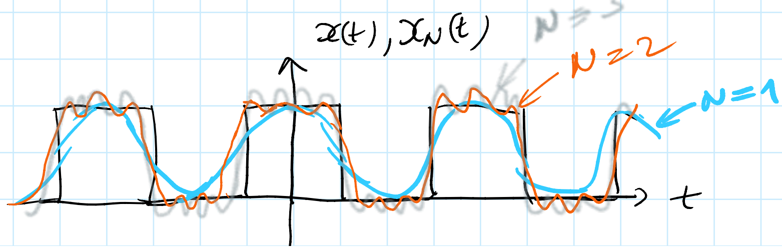
$$e_N(t) = x(t) - x_N(t)$$ GREŠKA

PERIODIČAN SIGNAL T_0

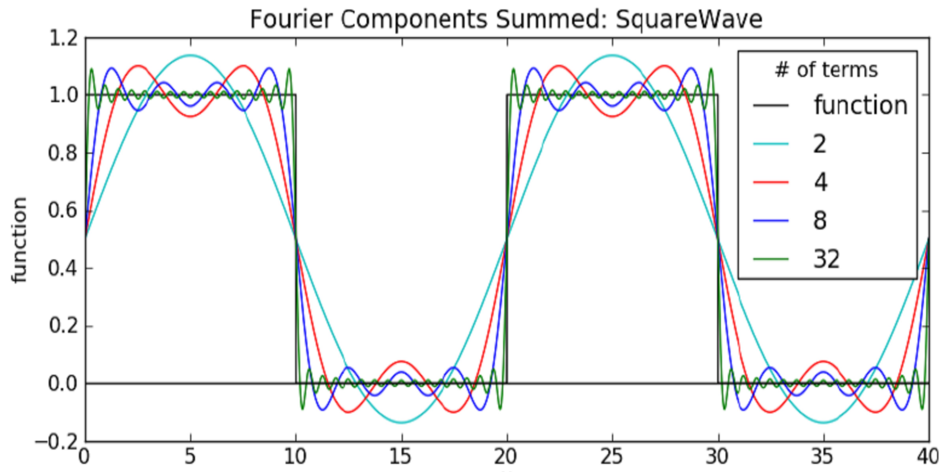
$$E_N = \int_{T_0} |e_N(t)|^2 dt$$ ENERGIJA SIGNALA GREŠKE

$$\lim_{N \to \infty} E_N = 0$$

- N-TA PARCIJALNA SUMA F REDA JE NAJBOLJA TRIGONOMETRIJSKA APROKSIMACIJA U SREDNJE KVADRATNOM SMISLU
- $x_N(t)$ $(N \in \{1, 2, \dots\})$ — $x_1(t), x_2(t) \dots$ NIJE RAVNOMERNO (UNIFORMNO) KONVERGENTAN U TAČKAMA PREKIDA FUNKCIJE $x(t)$



Fourier Components Summed: SquareWave



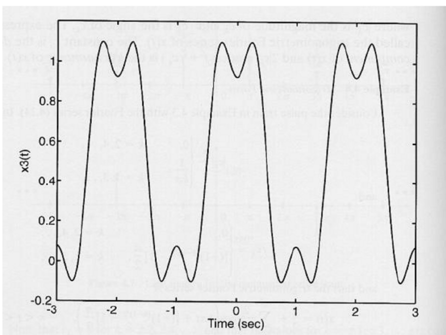


Figure 4.8 Plot of $x_N(t)$ when $N = 3$.

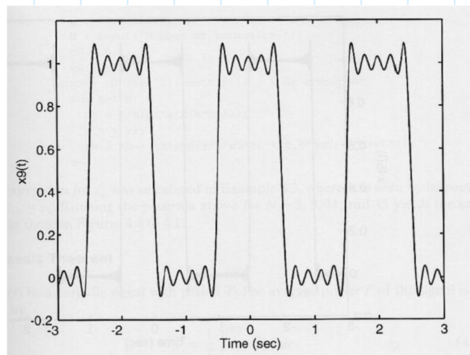


Figure 4.9 Approximation $x_9(t)$.

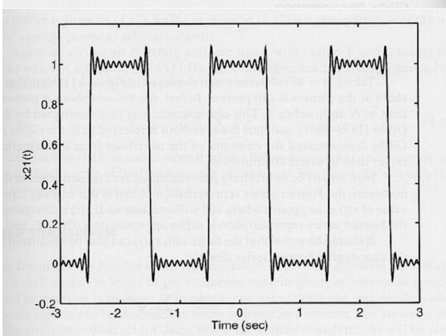


Figure 4.10 Approximation $x_{21}(t)$.

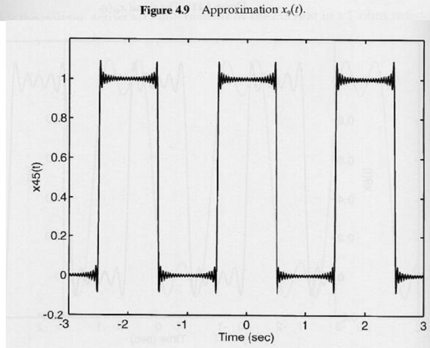
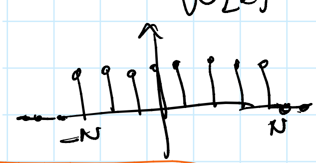


Figure 4.11 The signal $x_{45}(t)$.

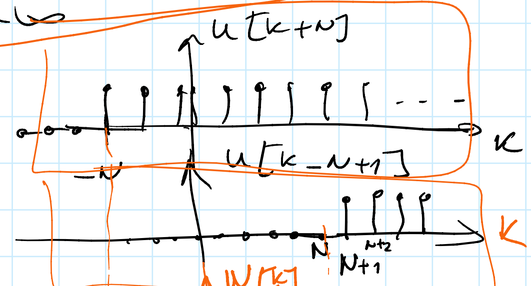
PROZORSKA FUNKCIJA

W[k]



$$\boxed{x_N(t)} = \sum_{k=-N}^{N} X[k]\, e^{jk\omega_0 t} = \boxed{\sum_{k=-\infty}^{+\infty} X[k]\cdot W[k]\, e^{jk\omega_0 t}}$$

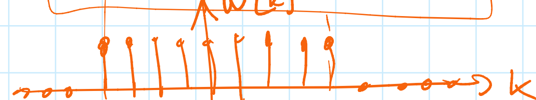
$$W[k] = \underbrace{u[k+N] - u[k-(N+1)]}$$



$$X_N[k] = X[k]\cdot W[k]$$

$$x_N(t) = \mathcal{F}_S^{-1}\{X[k]\cdot W[k]\}$$

$$x_N(t) = \frac{1}{T}\, x(t) \circledast W(t)$$



PROZORSKA FUNKCIJA

PROZORSKA FUNKCIJA

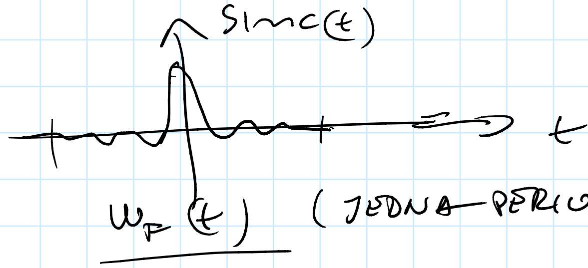
$$x_N(t) = \frac{1}{T_0}\, x(t) \circledast w(t)$$

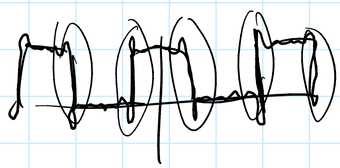
$$w(t) = \mathcal{F}_s^{-1}\{W[k]\} = \sum_{k=-\infty}^{+\infty} W[k]\, e^{jk\omega_0 t}$$

TABLIČKI KOEFICIJENT

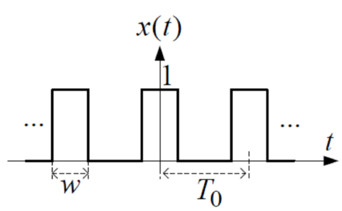
w(t) JE OBLIKA SINC FUNKCIJE

$$\mathcal{F}_s\left\{\mathrm{sinc}\left(\frac{t}{A}\right)\right\} = \frac{A}{T_0}\,\mathrm{rect}\left(k\frac{A}{T_0}\right)$$





OBLIK SIGNALA $x_N(t)$ POSLEDICA CIRKULARNE KONVOLUCIJE SIGNALA $x(t)$ I SIGNALA OBLIKA $\mathrm{sinc}(t)$

Osnovna perioda $\mathrm{rect}\left(\frac{t}{w}\right)$ 	$\frac{w}{T_0}\,\mathrm{sinc}\left(k\frac{w}{T_0}\right)$	$T_F = T_0$