Furijeovi redovi diskretnih signala

Saturday, May 8, 2021 6:00 PM

$$\phi_k[n] = e^{jk\Omega_F n} = e^{jk\frac{2\pi}{N_F}\cdot n}$$

$$e^{jk\Omega_F(n+m\cdot N_F)} = e^{jk\Omega_F\cdot n}\cdot e^{jk\Omega_F\cdot m\cdot N_F} =$$

$k\cdot m = p$

$$= e^{jk\Omega_F n}\cdot e^{jk\frac{2\pi}{\cancel{N_F}}\cdot m\cdot \cancel{N_F}} = e^{jk\Omega_F n}\cdot \underbrace{e^{2\pi\cdot j\cdot p}}_{1}$$

$$e^{2\pi j\cdot p} = \overbrace{\cos p\cdot 2\pi}^{1} + j\overbrace{\sin p\cdot 2\pi}^{0}$$

$\phi_k[n]$ JE N_F PERIODIČNO

$$\phi_k[n+m\cdot N_F] = \phi_k[n]$$

POSTOJI SAMO N_F RAZLIČITIH SEKVENCI $\phi_k[n]$

$$\sum_{n=n_1}^{n_1+N_F-1} e^{jk\Omega_F n} = \sum_{n=0}^{N_F-1} = \begin{cases} N_F & k=0,\pm N_F,\pm 2N_F\ldots \\ 0 & \text{ZA OSTALE } k\end{cases}$$

$$\sum_{n=0}^{N_F-1}\left(e^{jk\Omega_F}\right)^n = \frac{\left(e^{jk\Omega_F}\right)^{N_F}-1}{\underbrace{e^{jk\Omega_F}-1}_{\neq 0}} = \frac{\overbrace{e^{jk\frac{2\pi}{\cancel{N_F}}\cdot \cancel{N_F}}}^{1}-1}{e^{jk\Omega_F}-1} = 0$$

ZA $k\neq 0$

$$\boxed{\text{ZA } k=0 \quad \sum_{m=0}^{N_F-1} e^{0} = \sum_{n=0}^{N_F-1} 1 = N_F}$$

ZA KONT SIGNALE VAŽI

$$\int_{\tau}^{\tau+T_F} e^{jk\omega_F t}\,dt \qquad \int_{0}^{T_F} e^{jk\omega_F t}\,dt = \begin{cases} T_F & \text{ZA } k=0 \\ 0 & \text{ZA } k\neq 0\end{cases}$$

KOMPLEKSAN ↓

$x[n]$: $\underbrace{n_0 \le n \le n_0+N_F-1}_{ROI}$

$x_F[n] = x[n]$ NA SEGMENTU OD INTERESA

$x_F[n]$ „NE POSTOJI" VAN ROI

↑ NIZ OD N_F ČLANOVA!

N_F DIMENZIONI VEKTOR $\Rightarrow$ N_F DIMENZIONI VEKTORSKI PROSTOR

N_F DIMENZIONI VEKTOR $\Longrightarrow$ N_F DIMENZIONI VEKTORSKI PROSTOR

$\phi_k[n]$ — BAZIS VEKTORSKOG PROSTORA

$$x_F[n] = \sum_{k=m_0}^{m_0+N_F-1} X[k]\,\phi_k[n] = \sum_{k=m_0}^{m_0+N_F-1} X[k]\, e^{jk\Omega_F n}$$

KOEFICIJENTI U LINEARNOJ KOMBINACIJI (KOMPLEKSNI BROJEVI)

$\underline{C^n}$

$$\langle x_F[n], y_F[n] \rangle = \sum_{i=m_0}^{m_0+N_F-1} x_F[i] \cdot \overline{y_F[i]}$$

$\boxed{\bar{z} = z^*}$ KONJUGOVANO

$$\left.\begin{aligned} \langle \phi_k[n], \phi_k[n] \rangle &= N_F \\ \langle \phi_k[n], \phi_i[n] \rangle &= 0, \quad k \neq i \end{aligned}\right\}$$

$\phi_k[n]$ $\downarrow$ ORTOGONALNI BAZIS

$$\langle x_F[n], \phi_k[n] \rangle = \left\langle \sum_{i=m_0}^{m_0+N_F-1} X[i]\phi_i[n], \phi_k[n] \right\rangle =$$

$$= \sum_{i=m_0}^{m_0+N_F-1} X[i] \underbrace{\langle \phi_i[n], \phi_k[n] \rangle}_{N_F \text{ SAMO KAD JE } i=k} = N_F \cdot X[k]$$

$$X[k] = \frac{1}{N_F} \langle x_F[n], \phi_k[n] \rangle = \frac{1}{N_F} \sum_{n=m_0}^{m_0+N_F-1} x_F[n] \cdot \overline{\phi_k[n]}$$

$$= \boxed{X[k] = \frac{1}{N_F} \sum_{n=m_0}^{m_0+N_F-1} x_F[n]\, e^{-jk\Omega_F n}}$$

$$\boxed{\overline{\phi_k[n]} = e^{-jk\Omega_F n}}$$

$$\boxed{\Omega_F = 2\pi F_F = \frac{2\pi}{N_F}}$$

$x_F[n] = \sum \ldots$

DEFINISANO ZA $n \in \mathbb{Z}$

UNUTAR ROS

$x_F[n] = x[n]$

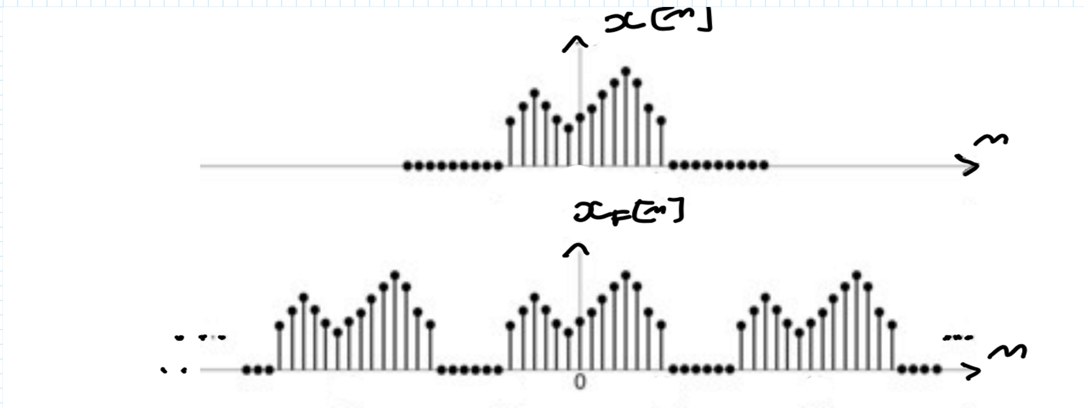
VAN ROS VAŽI JEDNAKOST AKO JE $x[n]$ PERIODIČNO SA N_F

INAČE NE VAŽI

AKO JE $x[n]$ PERIODIČNO SA N_F

INAČE NE VAŽI

$\underline{x_F[n]}$ ZA SVAKO n JE PERIODIČKI PRODUŽENO $x_F[n]$ DEFINISANO NA RoD



$X[k]$ — KOEFICIJENTI F. REDA SIGNALA $x[n]$ NA INTERVALU RoD

DISKRETAN SIGNAL U FREKVENCIJSKOM DOMENU

$\left.\begin{matrix} x[n] \\ X[k] \end{matrix}\right\}$ ISTI SIGNAL RAZLIČITA PREDSTAVA!

k — REDNI BROJ HARMONIKA OSNOVNE UČESTANOSTI Ω_F (F_F)

$X[0]$ — SREDNJU VREDNOST SIGNALA NA RoD

$\underline{x_F[n] = x[n]}$

↑ PERIODIČNO SA PERIODOM $N_0 \neq N_F$

$\left.\begin{matrix} N_F = m \cdot N_0 \\ m > 1 \end{matrix}\right\}$ ISTO KAO KOD FR KONTINUALNIH SIGNALA

SIGNALI KOJE RAZVIJAMO U FR: PERIODIČNI

$\underline{N_0, N_F}$ $\sum_{\langle N_0 \rangle}$, $\sum_{\langle N_F \rangle}$, $\sum_{l \in \langle N_0 \rangle}$

SKUP $X[k]$ — FREKVENCIJSKI SPEKTAR SIGNALA $x[n]$

$X[k] = |X[k]| e^{j\theta[k]}$ ← FAZNI SPEKTAR

$X[k] = |X[k]| e^{j\theta}$

FAZNI SPEKTAR

AMPLITUDSKI SPEKTAR

LINIJSKI SPEKTAR

$x[n]$ PERIODIČNO SA N_0

$X[k]$ PERIODIČNO SA N_0

$\underline{\phi_k[n]} = \underline{\phi[k][n]} = \Phi_{k,n}$ $N_0 \times N_0$

↑ MATRICA!

$x[n]$ DUŽINE N_0 $x[\cdot]$

$X[k]$ DUŽINE N_0 $X[\cdot]$

MATRIČNI RAČUN

↓

ALGEBRA MATRICA

DOS

+

FFT (DFT)

$x[n]$: REALAN

$$X[-k] = X^*[k] = \overline{X[k]}$$

$|X[k]| = |X[-k]|$ AMPLITUDSKI SPEKTAR PARAN!

$\theta[-k] = -\theta[k]$ FAZNI SPEKTAR NEPARAN

ISTE OSOBINE VEZANE ZA PARNOST I NEPARNOST KAO U SLUČAJU $F_R\{x(t)\}$

△ PRIMER

$$x[n] = \cos \Omega_0 n$$

a) $\Omega_0 = \pi/3$

b) $\Omega_0 = \pi\sqrt{2}$

a) $\Omega_0 = \frac{\pi}{3} = 2\pi \cdot F_0$

$F_0 = \frac{\Omega_0}{2\pi} = \frac{\pi}{3} \cdot \frac{1}{2\pi} = \frac{1}{6} = \frac{1}{N_0}$

$N_0 = 6$

$0 \leq k \leq 5$

$0 \leq n \leq 5$

$\left.\begin{matrix} x[n] \\ X[k] \end{matrix}\right\} N_0 = 6$

$$X[k] = \frac{1}{N_0} \sum_{n=0}^{5} x[n] e^{-jk\Omega_0 n}$$

$$x[n] = \cos \Omega_0 n = \frac{1}{2} e^{j\Omega_0 n} + \frac{1}{2} e^{-j\Omega_0 n}$$

$$X[k] = \frac{1}{N_0} \sum_{n=0}^{5} \left(\frac{1}{2} e^{j\Omega_0 n} + \frac{1}{2} e^{-j\Omega_0 n} \right) \cdot e^{-jk\Omega_0 n}$$

5

$$= \frac{1}{2} \cdot \left(\underbrace{\frac{1}{N_0} \sum_{n=0}^{5} e^{j\Omega_0 n(1-k)}}_{=1} + \underbrace{\frac{1}{N_0} \sum_{n=0}^{5} e^{-j\Omega_0 n(1+k)}}_{=1} \right)$$

$1-k=0$

$k=1$

$1+k=0$

$k=-1$

$1+k=N_0$

$k=N_0-1=5$

$0 \le k \le 5$

$X[k] = \frac{1}{2}$ ZA $k=1$ i ZA $k=5$

$X[k]=0$ ZA OSTALO

$$X[k] = \frac{1}{2} \delta[k-1] + \frac{1}{2} \delta[k-5]$$

$$X[k] = \frac{1}{2} \delta[k-1] + \frac{1}{2} \delta[k+1] \leftarrow$$

ASOCIRA NA $F_R\{\cos \omega_0 t\}$

$X[k] = X[k \pm m N_0]$ ZBOG PERIODIČNOSTI

$$x[n] = \cos \Omega_0 n = \frac{1}{2} e^{j \cdot 1 \Omega_0 n} + \frac{1}{2} e^{j(-1) \cdot \Omega_0 n}$$

$k=1$ $k=-1$

$X[k=1]$ $X[k=-1]$

$$X[-1] = X[-1+N_0] = X[-1+6] = X[5]$$

$$0 \le k \le 5$$

b) $x[n] = \cos \Omega_0 n$

$\underbrace{\Omega_0 = \pi\sqrt{2}}_{\text{NIJE PERIODIČAN}}$

$\Omega_0 = 2\pi F_0$

$$F_0 = \frac{\Omega_0}{2\pi} = \frac{\cancel{\pi}\sqrt{2}}{2\cancel{\pi}} = \frac{\sqrt{2}}{2} \cancel{= \frac{1}{N_0}}$$

CEO BROJ

1 PRIMER

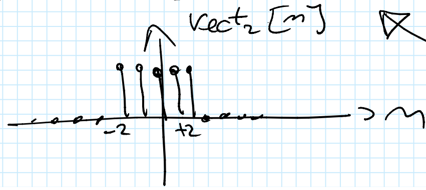
APERIODIČNO $N_0 = 8$

$x[n] \leftarrow$ PERIODIČNO

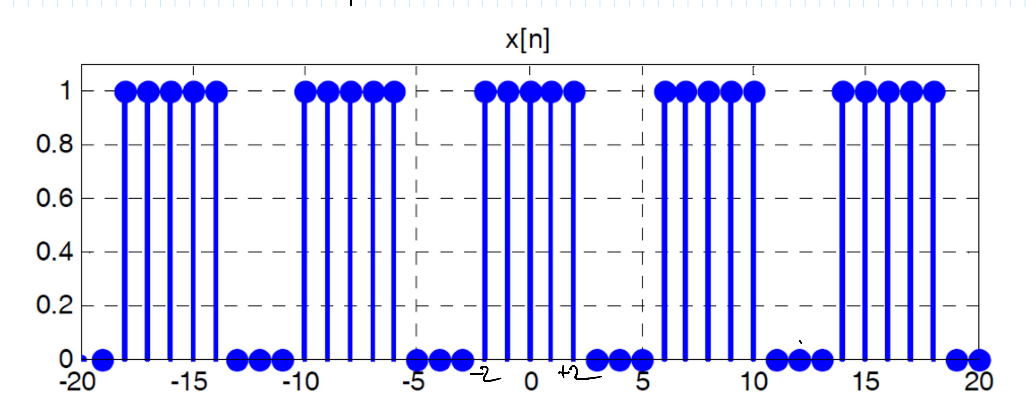
1 PRIMER

APERIODIČNO $\quad N_0 = 8$

$$x[n] = \text{rect}_2[n] * Ш_8[n] \leftarrow \text{PERIODIČNO}$$



JEDINIČNI IMPULS NA SVAKOM 8. MESTU



$x[n]: \; N_0 = 8$

$$X[k] = \frac{1}{N_0} \sum_{n=\langle N_0 \rangle} x[n] e^{-jk\Omega_0 n} = \frac{1}{8} \sum_{n=-2}^{2} 1 \cdot e^{-jk\Omega_0 n}$$

$\omega_0 \; t$

$$X[k] = \frac{1}{8} + \frac{1}{4} \cdot \sum_{n=1}^{2} \frac{1}{2} \underbrace{\left(e^{-jk\Omega_0 n} + e^{+jk\Omega_0 n}\right)}_{\cos k\Omega_0 n} \qquad \Omega_0 = \frac{2\pi}{N_0} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$= \frac{1}{8} + \frac{1}{4}\cos 2k\Omega_0 + \frac{1}{4}\cos k\Omega_0 = \boxed{\frac{1}{8} + \frac{1}{4}\cos k\cdot\frac{\pi}{2} + \frac{1}{4}\cos k\cdot\frac{\pi}{4}}$$

$x[n], y[n] \; .. \; N_0, N_F \qquad \boxed{n_0 \neq N_0} \qquad \mathcal{F}_R\{x[n]\} = X[k], \quad \mathcal{F}_R\{y[n]\} = Y[k]$

1) LINEARNOST

2) $\mathcal{F}_s\{x[n-n_0]\} = e^{-jk\Omega_0 n_0} X[k]$

3) $\mathcal{F}_s\{x[n] \cdot e^{jk_0\Omega_0 n}\} = X[k-k_0]$

$\mathcal{F}_R\{x(t)\}$

t, ω_0

4) INVERZIJA VREMENSKE OSE

$$\mathcal{F}_s\{x[-n]\} = X[-k]$$

5) $\mathcal{F}_s\{\overline{x[n]}\} = \overline{X[-k]}$

6) RAZVOJ PROIZVODA DVA SIGNALA

$x[n], \; y[n], \; N_0$ ZA OBA $\quad X[k], Y[k]$

6) RAZVOJ ...

$x[n]$, $y[n]$, N_0 ZA OBA $X[k], Y[k]$

$$z[n] = x[n] \cdot y[n]$$

↑ PERIODIČAN SA N_0

$$Z[k] = \sum_{\ell \in \langle N_0 \rangle} Y[\ell]\, X[k-\ell] \quad \text{(CIRKULARNA KONVOLUCIJA)}$$

$$Z[k] = Y[k] \circledast X[k] \quad \left(\text{ISTO KAO KOD } \mathcal{F}_s\{x(t) \cdot y(t)\} \right)$$

7) FURIJEOV RED CIRKULARNE KONVOLUCIJE

$$z[n] = x[n] \circledast y[n]$$

$$Z[k] = N_0\, Y[k] \cdot X[k]$$

PROIZVOD $\rightleftarrows$ KONVOLUCIJA

CIRKULARNA U OBA DOMENA

8) RAZVOJ PRVE DIFERENCE UNAZAD (UNAPRED)

$$\mathcal{F}_s\{\nabla x[n]\} = \mathcal{F}_s\{x[n] - x[n-1]\} =$$

$$\mathcal{F}_s\{x[n]\} - \mathcal{F}_s\{x[n-1]\} = X[k] - e^{-jk\Omega_0} X[k] =$$

$$= X[k]\left(1 - e^{-jk\Omega_0}\right)$$

$$\mathcal{F}_s\{\Delta x[n]\} = X[k]\left(e^{+jk\Omega_0} - 1\right)$$

9) RAZVOJ AKUMULACIJE

$$z[n] = \sum_{m=-\infty}^{n} x[m] \leftarrow \text{AKUMULACIJA}$$

PERIODIČAN SA N_0

$X[0] \neq 0$ $\quad x[n]$ IMA SREDNJU VREDNOST

SUMA (RED) DIVERGIRA

$\boxed{X[0] = 0}$ IMA SMISLA SUMIRANJE

$$x[n] = z[n] - z[n-1] = \nabla z[n] \quad / \mathcal{F}_s$$

NA LEVU I DESNU STRANU

$$X[k] = Z[k]\left(1 - e^{-jk\Omega_0}\right)$$

$$Z[k] = \frac{X[k]}{\underbrace{1 - e^{-jk\Omega_0}}_{\neq 1}}$$

10) ~~PASEVALOVA TEOREMA~~

$$\frac{1}{N_0}\sum_{n\in\langle N_0\rangle}|x[n]|^2 = \sum_{k\in\langle N_0\rangle}|X[k]|^2$$

11) $x[n]$ — ~~PARAN~~ $x[n] = x[-n]$

$$X[k] = X[-k]$$

$x[n]$ — NEPARAN $x[n] = -x[-n]$

$$X[k] = -X[-k]$$

XE

$x[n]$ i REALAN + PARAN

$X[k]$ PARNI, REALNI

$x[n]$ REALAN i NEPARAN

$X[k]$ NEPARNI i IMAGINARNI

DOKAZIVATI ~~ZA~~ KONTINUALNE SIGNALE!

~~ZA~~ REALNE SIGNALE $X[k] = \overline{X[-k]}$

$$\mathcal{F}_s\{x[-n]\} = X[-k]$$

PITANJE KONVERGENCIJE

NE POSTOJI : SUME SU KONAČNE!