Odsek za elektroniku
Signali i sistemi – popravni I kolokvijum resenja 14.6.2019.

1. Dati su razni sitemi i signali.

a) [5] Za je signal $h(t) = -1 + 2\sum_{k=-\infty}^{\infty} \mathrm{rect}(t-2k)$ izračunati srednju snagu P, i energiju W. P=1;W=∞

b) [5] Ako je $x[n] = a^n \mathrm{u}[n]$, a $h[n] = b^n \mathrm{u}[n]$, tada je $h[-n] * x[-n] = y[-n] = \dfrac{a^{-n+1} - b^{-n+1}}{a-b}\mathrm{u}[-n]$

c) [6] Ako je $y[n] = 2x[n] + \sum_{k=-\infty}^{\infty} x[n-k+2]\delta[k-1]$ i $x[n] = 2^n \mathrm{u}[n+1]$ tada je

$$y[n] = 2x[n] + \sum_{k=-\infty}^{\infty} x[n+2-k]\delta[k-1] = 2x[n] + x[n+2] * \delta[n-1] = 2x[n] + x[n+1] =$$

$$2\cdot 2^n \mathrm{u}[n+1] + 2^{n+1}\mathrm{u}[n+2] = 2\cdot 2^{n+1}\mathrm{u}[n+1] + 2^{n+1}\delta[n+2] = 2^{n+2}\mathrm{u}[n+1] + 2^{-1}\delta[n+2]$$

d) [3] Ako je dužina signala $x[n]$ jednaka N_x i ako je dužina signala $x[n]*x[n]*x[n]$ jednaka 28, tada je N_x =10

c) [3] c) [3] Ako je E$y[n]$ = $x[n + 1] + 4\,x[n + 4]$ - D$x[n - 2]$ => $y[n]$ = $x[n] + 4\,x[n + 3] - x[n - 4]$
tada je impulsni odziv $h[n]$= $\delta[n] + 4\,\delta[n + 3] - \delta[n - 4]$

d) [3] Ako je $x[n]=n(n+1)$ a impulsni odziv $h[n] = \mathrm{u}[n]-\mathrm{u}[n-2]$ odrediti odziv $y[n]$ u formi $y[n]=An^2+Bn+C$
A=2, B=0, i C=0.

e) [5] Ako je $x(t) = e^{-t}(\mathrm{u}(t) - \mathrm{u}(t-1))$ tada je njegova Furijeova transformacija $X(j\omega) =$

$$x(t) = e^{-t}\mathrm{u}(t) - e^{-(t-1)}\mathrm{u}(t-1)e^{-1} \Rightarrow X(s) = \frac{1-e^{-(s+1)}}{s+1}; s = j\omega$$

f) [5] Na osnovu prethodne tačke odrediti Furijeovu transformaciju signala: $g(t) = x(t)\mathrm{u}(t) + x(-t)\mathrm{u}(-t)$:

$$G(s) = \underbrace{X(s) + X(-s)}_{5\,bodova!} = 2\frac{1-e^{-1}\cos\omega - \omega e^{-1}\sin\omega}{1+\omega^2}$$

2.
Resenje a)

$$2h_1[n] + 11h_1[n-1] + 17h_1[n-2] + 6h_1[n-3] = \delta[n] \to P(\lambda) = 2(\lambda+2)(\lambda+3)(\lambda+1/2)$$

$$h_1[n] = \left(A(-2)^n + B(-3)^n + C(-1/2)^n\right)\mathrm{u}[n]$$

$$h_1[0] = 1/2, h_1[1] = -11/4, h_1[2] = 87/8$$

$$\left.\begin{array}{l} 1/2 = A+B+C \Rightarrow 2A+2B+2C = 1 \\ -11/4 = -2A-3B-C/2 \Rightarrow 8A+12B+2C = 11 \\ 87/8 = 4A+9B+C/4 \Rightarrow 32A+72B+2C = 87 \end{array}\right\} A = -4/3, B = 9/5, C = 1/30$$

$$h[n] = 2h_1[n] + h_1[n+1] = \left(\frac{3}{5}(-3)^{n+1} - \frac{1}{10}(-1/2)^{n+1}\right)\mathrm{u}[n+1]$$

b) $\underbrace{x[n] = \mathrm{u}[n-1]}_{n>2}$, $x[0] = x[1] = 0, x[2] = 1 \to x[n] = \mathrm{u}[n-2], \forall n$; (prihvata se i prekršteno $x[0] \to y[0]...$)

$$y_{us}[n-3] = (2\cdot 1^n + 1^n)/P(1) = 1/12 = y_{us}[n]$$

Za preinicijalne uslove =0:

$$y[n]=y_P[n]=h[n]*x[n]=\left(\frac{3}{5}(-3)^{n+1}-\frac{1}{10}(-1/2)^{n+1}\right)\mathrm{u}[n+1]*\mathrm{u}[n-2]=$$

$$\left(\frac{3}{5}(-3)^{n}-\frac{1}{10}(-1/2)^{n}\right)\mathrm{u}[n]*\mathrm{u}[n-1]=D\left(\left(\frac{3}{5}(-3)^{n}-\frac{1}{10}(-1/2)^{n}\right)\mathrm{u}[n]*\mathrm{u}[n]\right)=$$

$$\left(\frac{3}{5}\frac{(-3)^n-1}{-3-1}-\frac{1}{10}\frac{(-1/2)^n-1}{-1/2-1}\right)\mathrm{u}[n-1]$$

$$y_{PR}[n]=\left(-\frac{3}{5}\frac{(-3)^n}{4}+\frac{2}{10}\frac{(-1/2)^n}{3}\right)\mathrm{u}[n-1]$$

3. a) [15]Primenom tablica i teorema odrediti koeficijente $A[k]$ i $B[k]$ razvoja u trigonometrijski furijeov red signala $x(t)=8/(10+6\cos 4t)$. c) [5] Izračunati integral $\int_{1.33}^{1.33+\pi/2} x(t)\mathrm{d}t=$ ____________ . d) [5] Izračunati integral $\int_{-\pi/2}^{0} x^2(t)\mathrm{d}t=$ ____________

Resenje: a)

$$\frac{1-a^2}{1-2a\cos\omega t+a^2}=1+2\sum_{k=1}^{\infty}a^k\cos k\omega t \Rightarrow \mathit{paran\,signal}: B[k]=0 \text{ [5 bodova]}$$

[ostalo 10 bodova]

$$A(1+a^2-2a\cos\omega t)=10+6\cos 4t$$

$$\omega=4, T=\pi/2$$

$$A(1+a^2)=10$$

$$-2Aa=6 \Rightarrow A=-3/a \Rightarrow \frac{1+a^2}{a}=-\frac{10}{3}\Rightarrow \frac{1}{a}+a=-3-\frac{1}{3}\Rightarrow a=-\frac{1}{3}, A=9$$

$$B\cdot(1-a^2)=B\cdot\frac{8}{9}=8\Rightarrow B=9$$

$$x(t)=\frac{8}{10+6\cos 4t}=\frac{B}{A}\cdot\frac{1-a^2}{1-2a\cos\omega t+a^2}=1+2\sum_{k=1}^{\infty}(-3)^{-k}\cos 4kt$$

$$A[0]=1;\quad A[k]=2\cdot(-3)^{-k}$$

b) ,c)

$$\frac{1}{T}\int_{-T}^{0}x^2(t)\mathrm{d}t=1^2+2^2\sum_{1}^{\infty}\left(\frac{1}{\sqrt{2}}\left(-\frac{1}{3}\right)^k\right)^2=1^2+2\sum_{1}^{\infty}\left(\frac{1}{9}\right)^k=1+2\frac{1/9}{1-1/9}=5/4$$

$$\int_{-T}^{0}x^2(t)\mathrm{d}t=\frac{5\pi}{8}$$

$$\frac{1}{T}\int_{c}^{c+T}x(t)\mathrm{d}t=A[0]=1\Rightarrow\int_{c}^{c+T}x(t)\mathrm{d}t=T$$

4. Dat je signal $x(t)=\mathrm{rect}(t/2T)$, T=30s

a) [10] Odrediti analitički izraz za fazni spektar signala $x(t)$ i $x(t+1)$:

b) [10] Odrediti analitički frekvencijski spektar signala $y(t)=x(t)\cdot\cos(t)\cdot\cos(6t)$ i nacrtati amplitudski spektar.

Resenje: a)

Na osnovu tablice

$$F\{x(t)\}=2T\frac{\sin(\omega T)}{\omega T}\qquad F\{x(t+1)\}=2T\frac{\sin(\omega T)}{\omega T}e^{j\omega\cdot 1}$$

$$\theta(\omega) = \arg\{2T\frac{\sin(\omega T)}{\omega T}\} = \left(\operatorname{sgn}\left(\frac{\sin(\omega T)}{\omega T}\right) - 1\right)\frac{\pi}{2} \text{ i } \theta_1(\omega) = \arg\{2T\frac{\sin(\omega T)}{\omega T}e^{j\omega\cdot 1}\} = \left(\operatorname{sgn}\left(\frac{\sin(\omega T)}{\omega T}\right) - 1\right)\frac{\pi}{2} + \omega$$

b) u pitanju je "dvostruka" modulacija

$$Y(j\omega) = \frac{1}{4}\left(X(j\omega - 5j) + X(j\omega + 5j) + X(j\omega - 7j) + X(j\omega + 7j)\right)$$

Na grafiku se vide uski Sinc signali amplitude ¼ na pozicijama ±5 i ±7