Odsek za elektroniku
Signali i sistemi – II kolokvijum resenja 14.6.2019.

1. a) [6] $F\left\{e^{j\Omega_0 n}\right\} = 2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2k\pi)$ , $F\left\{\sin(\Omega_0 n)\right\} = \frac{\pi}{j} \sum_{k=-\infty}^{\infty} \left(\delta(\Omega - \Omega_0 - 2k\pi) - \delta(\Omega + \Omega_0 - 2k\pi)\right)$

b) [5]

$x[0] = 0 \Rightarrow X(z) = x[1]z^{-1} + x[2]z^{-2} + ... \Rightarrow zX(z) = x[1] + x[2]z^{-1} + ...$

$\lim_{z\to\infty} zX(z) = x[1] = 1$

c) [5] Ako je $L\{x(t)\} = X(s), ROC(x) = \mathrm{Re}\{s\} \subset R$ tada za $y(t) = 2x(t/3) + 3x(-2t)$ važi

$L\{x(t/3)\} = 3X(3s), ROC = 3ROC(x)$

$L\{x(-2t)\} = \frac{1}{2} X\left(-\frac{s}{2}\right), ROC = -\frac{1}{2} ROC(x)$

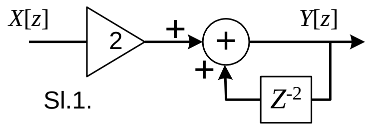
Ukoliko postoji $-\frac{1}{2} ROC(x) \cap 3ROC(x)$ tada je

$Y(s) = 6X(3s) + \frac{3}{2} X\left(-\frac{s}{2}\right), ROC(y) \subseteq -\frac{1}{2} ROC(x) \cap 3ROC(x)$

d) [4] Ako je $L\{x(t)\} = X(s), ROC(x) = \mathrm{Re}\{s\} \subset R$ tada za $y(t) = e^{5t} x(t)$ važi

$Y(s) = X(s-5)$ , i $ROC(y) = ROC(x) + 5$

e) Na slici 1 je dat blok dijagram diskretnog sistema:
[6] Prenosna funkcija i impulsni odziv datog sistema glase



Sl.1.

$H(z) = 2\frac{1}{1 - z^{-2}} = \frac{2z^2}{z^2 - 1} = \frac{z}{z-1} + \frac{z}{z+1}$

$h[n] = \left(1 + (-1)^n\right) u[n]$

f) [4] Za sistem iz prehodne tačke važi da je u opsegu $|\Omega| \le \pi$ amplitudska karakteristika sistema jednaka funkciji $f(\sin(k\cdot\Omega))$ . Tada su amplitudska i fazna karakteristika sistema dati izrazima

$k = 1, \quad |H(j\Omega)| = 1/|\sin(\Omega)| \quad \arg\{H(j\Omega\} = \Omega - \pi/2, \quad \textit{za osnovni period } |\Omega| \le \pi$ .

2. Na ulaz prati-pamti kola dovodi se signal $x(t) = \cos(t)$ , dok se na kontrolni priključak dovode kratkotrajni impulse, deset po periodi (alternativno na svakih 125ms) , tako da se vreme praćenja moze zanemariti. Napon na izlazu kola je $x_s(t)$

a) [5] Napisati izraz za napon na izlau kola i nacrtati ga za $0 < t < 2\pi$ .

$T_0 = 2\pi, \quad T_S = T_0/10 = 2\pi/10, \quad \omega_S = 2\pi/T_S = 10$

(napomena: slicno za $T_S = 125ms$, slika je slozenija)

$x_s(t) = \sum_{k=-\infty}^{\infty} \cos(kT_S)\left(\mathrm{u}(t - kT_S) - \mathrm{u}(t\text{-}(k+1)T_S)\right)$

b) [5] $X_s(j\omega) = \underbrace{\frac{1 - e^{-j\omega T_S}}{j\omega}}_{H_{SH}(j\omega)} \sum_{k=-\infty}^{\infty} x(kT_S) e^{-jk\omega T_S} = H_{SH}(j\omega) \frac{1}{T_S} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_S))$

$X(j\omega) = j\pi\left(\delta(\omega+1) - \delta(\omega-1)\right) \Rightarrow X_S(j\omega) = \frac{\pi}{T_S} \sum_{k=-\infty}^{\infty} H_{SH}(j\omega)\left(\delta(\omega + 1 - k\omega_s) + \delta(\omega - 1 - k\omega_s)\right)$

$X_S(j\omega) = 5 \sum_{k=-\infty}^{\infty} H_{SH}(j\omega)\left(\delta(\omega + 1 - 10k) + \delta(\omega - 1 - 10k)\right)$

c) [10]

Signal $X_S(j\omega)$ sadrži δ impulse na učestanostima ±1, 10±1, –10±1... itd dok signal $Y(j\omega)$ sadrži δ impulse na učestanostima ±2. Prema tome nije moguće izdvojiti takva dva impulsa iz signala $X_S(j\omega)$ jer ih on ne sadrži, pa traženi filter ne postoji.

3.

[10 ]

$$2h[n]+11h[n-1]+17h[n-2]+6h[n-3]=2\delta[n]+\delta[n+1]$$

$$2H(z)+11H(z)z^{-1}+17H(z)z^{-2}+6H(z)z^{-3}=2+z$$

$$P(z)=2(z+2)(z+3)(z+1/2)$$

$$P(z)H(z)=(2+z)z^3 \Rightarrow H(z)=\frac{(2+z)z^3}{2(z+2)(z+3)(z+1/2)}=\frac{z^3}{2(z+3)(z+1/2)}=\frac{3}{5}\frac{z}{z+3}z-\frac{1}{10}\frac{z}{z+1/2}z$$

$$h[n]=\left(\frac{3}{5}(-3)^{n+1}-\frac{1}{10}(-1/2)^{n+1}\right)\mathrm{u}[n+1]$$

$$2h_1[n]+11h_1[n-1]+17h_1[n-2]+6h_1[n-3]=\delta[n]$$

$$2H(z)+11H(z)z^{-1}+17H(z)z^{-2}+6H(z)z^{-3}=1$$

$$P(z)=2(z+2)(z+3)(z+1/2)$$

$$P(z)H_1(z)=z^3 \Rightarrow H_1(z)=\frac{z^3}{2(z+2)(z+3)(z+1/2)}=\frac{9}{5}\frac{z}{z+3}+\frac{1}{30}\frac{z}{z+1/2}-\frac{4}{3}\frac{z}{z+2}$$

$$h_1[n]=\left(\frac{9}{5}(-3)^n+\frac{1}{30}(-1/2)^n-\frac{4}{3}(-2)^n\right)\mathrm{u}[n]$$

$$h[n]=2h_1[n]+h_1[n+1]=\left(\frac{3}{5}(-3)^{n+1}-\frac{1}{10}(-1/2)^{n+1}\right)\mathrm{u}[n+1]$$

a) [15] $\underbrace{x[n]=\mathrm{u}[n-1]}_{n>2}$, $x[0]=x[1]=0,\ x[2]=1 \to x[n]=\mathrm{u}[n-2], \forall n$;

(napomena : prihvata se i “prekršteno” $x[0]\to y[0]...$)

sopstveni i prinudni odziv bez razdvajanja na ustaljenji u prelazni nosi 12 bodova)

$$Y_P(z)=H(z)X(z)=\frac{z^3}{2(z+3)(z+1/2)}\left(\frac{z^{-1}}{z-1}\right)=\frac{z^2}{2(z+3)(z+1/2)(z-1)}=$$

$$=\frac{3}{20}\frac{z}{z+3}-\frac{1}{15}\frac{z}{z+1/2}+\frac{1}{12}\frac{z}{z-1}$$

$$y_P[n]=\underbrace{\left(\frac{3}{20}(-3)^n-\frac{1}{15}(-1/2)^n\right)\mathrm{u}[n]}_{ostatak}+\underbrace{\frac{1}{12}\mathrm{u}[n]}_{y_{US}}$$

Za nulte preinicijalne uslove ostatak je prelazni odziv [15]

Za preinicijalne uslove <> 0:

Kako je $H(z)=\dfrac{z^3}{2(z+3)(z+1/2)}$ to je $y_S[n+2]+\dfrac{7}{2}y_S[n+1]+\dfrac{3}{2}y_S[n]=0$

$$z^2Y_S(z)-z^2y[0]-zy[1]+\frac{7}{2}zY_S(z)-\frac{7}{2}zy[0]+\frac{3}{2}Y_S(z)=0$$

$$(z+3)(z+1/2)Y_S(z)=z^2y[0]+z\left(y[1]+7y[0]/2\right)$$

$$Y_S(z)=z^2\frac{1}{(z+3)(z+1/2)}y[0]+z^2\frac{1}{(z+3)(z+1/2)}z^{-1}\left(y[1]+7y[0]/2\right)$$

$$y_s[n]=y[0]\left(-\frac{6}{5}(-1/2)^n+\frac{6}{5}(-3)^n\right)\mathrm{u}[n]+\left(y[1]+7y[0]/2\right)\left(-\frac{6}{5}(-1/2)^{n-1}+\frac{6}{5}(-3)^{n-1}\right)\mathrm{u}[n-1]$$

$$y_{PR}[n] = \underbrace{\left(\frac{9}{10}(-3)^{n-1} - \frac{1}{15}(-1/2)^{n-1}\right)}_{ostatak} \mathrm{u}[n-1] + y_S[n]$$

4. [25]

$$h[n] = (-3+5^{-n})\mathrm{u}[n] \Rightarrow H(z) = -3\frac{z}{z-1} + \frac{z}{z-1/5};\ ROC_H : \rho > 1$$

$$x[n] = -2^n \mathrm{u}[-n-1] \Rightarrow X(z) = \frac{z}{z-2};\ ROC_x : \rho < 2$$

$$X(z)H(z) = -3\frac{z}{z-1}\frac{z}{z-2} + \frac{z}{z-1/5}\frac{z}{z-2};\ ROC_{HX} : 1 < \rho < 2$$

$$X(z)H(z) = -3\left(2\frac{z}{z-2} - \frac{z}{z-1}\right) + \frac{1}{9}\left(10\frac{z}{z-2} - \frac{z}{z-1/5}\right)$$

$$y[n] = -3\left(2\cdot\left(-2^n \mathrm{u}[-n-1]\right) - \mathrm{u}[n]\right) + \frac{1}{9}\left(10\left(-2^n \mathrm{u}[-n-1]\right) - (1/5)^n \mathrm{u}[n]\right)$$