Signali i sistemi – kolokvijum 2 i integralni ispit 04.07.2018.

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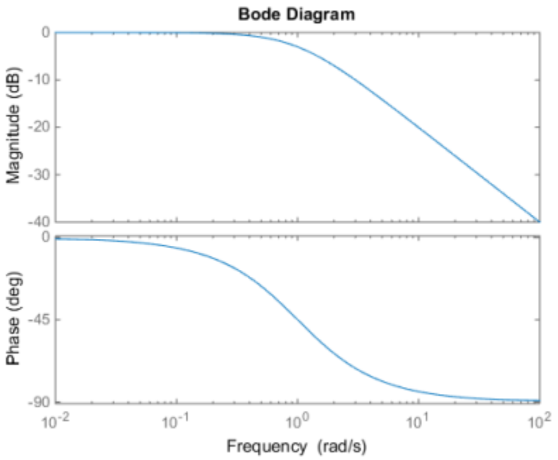
Kolokvijum 2

1. zadatak (45 poena) – na kraju

2. zadatak (25 poena)

a)



b) $h(t) = e^{-t}\mathrm{u}(t)$

c) $H(j\Omega) = \dfrac{e^{j\Omega}+1}{2001\cdot e^{j\Omega}-1999} = \dfrac{1}{2001}\left(1+\dfrac{4000}{2001}\cdot\dfrac{e^{-j\Omega}}{1-\dfrac{1999}{2001}\cdot e^{-j\Omega}}\right)$

Teorema o odabiranju je ispoštovana.

d) $h[n] = \dfrac{1}{2001}\delta[n] + \dfrac{4000}{2001^2}\cdot\left(\dfrac{1999}{2001}\right)^{n-1}\mathrm{u}[n-1]$

3. zadatak (20 poena)

a)

$H(s) = \dfrac{5}{(s+3)(s+2)(s+1)} = \dfrac{5/2}{s+3} + \dfrac{-5}{s+2} + \dfrac{5/2}{s+1}$

Pol $s=-3$ pripada kauzalnom delu, a $s=-2$ i $s=-1$ antikauzalnom delu.

$h(t) = \dfrac{5e^{-3t}}{2}\mathrm{u}(t) + 5e^{-2t}\mathrm{u}(-t) - \dfrac{5e^{-t}}{2}\mathrm{u}(-t)$

b) ROC: $\dfrac{3}{4} < |z| < 1$

4. zadatak (20 poena)

a) Signal $x[n]$ predstavlja konvoluciju $x[n] = n^2 a^n \mathrm{u}[n] * \mathrm{u}[n]$. Kako je $\mathcal{Z}\{\mathrm{u}[n]\} = \dfrac{z}{z-1}$,

$\mathcal{Z}\{n\mathrm{u}[n]\} = \dfrac{z}{(z-1)^2}$, $\mathcal{Z}\{n^2\mathrm{u}[n]\} = \dfrac{z(z+1)}{(z-1)^3}$ i $\mathcal{Z}\{a^n n^2\mathrm{u}[n]\} = \dfrac{az(z+a)}{(z-a)^3}$, računa se:

$$X(z) = \mathcal{Z}\{n^2 a^n \mathrm{u}[n]\}\mathcal{Z}\{\mathrm{u}[n]\} = \frac{az^2(z+a)}{(z-a)^3(z-1)}.$$

b) Kako je $\lim\limits_{z\to\infty} Y(z) = y[0] = 5$, računa se

$$Y(z)=\frac{zy[0]}{z+1}+\frac{z^2\left(z+1\right)}{\left(z-1\right)^4\left(z+1\right)}=\frac{zy[0]}{z+\frac{1}{2}}+\frac{z^2}{\left(z-1\right)^4}.$$

$$y[n]=\left(5\cdot(-1)^n+\binom{n+1}{3}\right)\mathrm{u}[n]$$

5. zadatak (30 poena)

b)

$$(D-1)(D-2)^2y[n]=(D^3-5D^2+8D-4)y[n]$$
$$-4y[n]+8y[n-1]-5y[n-2]+y[n-3]=3x[n-2]$$

Početni uslovi impulsnog odziva:

$n=0$

$h[0]=-3\delta[-2]=0 \;\Rightarrow\; h[0]=0$

$n=1$

$-4h[1]+8h[0]=3\delta[-1]=0 \;\Rightarrow\; h[1]=0$

$n=2$

$-4h[2]+8h[1]-5h[0]=3\delta[0]=3 \;\Rightarrow\; h[2]=-\frac{3}{4}$

$$h[n]=\left(A+\left(B+nC\right)\cdot 2^{-n}\right)\mathrm{u}[n]$$

$$-A=B=C=3$$

c) $a^n\mathrm{u}[n]*b^n\mathrm{u}[n]=\frac{a^{n+1}-b^{n+1}}{a-b}\mathrm{u}[n]$, $a^n\mathrm{u}[n]*\mathrm{u}[n]=\frac{a^{n+1}-1}{a-1}\mathrm{u}[n]$, $\mathrm{u}[n]*\mathrm{u}[n]=(n+1)\mathrm{u}[n]$

$$na^n\mathrm{u}[n]*b^n\mathrm{u}[n]=\left(\frac{(n+1)a^{n+1}}{a-b}-a\frac{a^{n+1}-b^{n+1}}{(a-b)^2}\right)\mathrm{u}[n]$$

$$h[n]*x[n]=D\left(\left(A+\left(B+nC\right)\cdot 2^{-n}\right)\mathrm{u}[n]*\left(5+3^n\right)\mathrm{u}[n]\right)=$$
$$=-15\left(n-4+2^{-(n-1)}(n+2)\right)\mathrm{u}[n-1]+3\left(-\frac{3^n}{50}-\frac{2n\cdot 2^{-n}}{25}-\frac{22}{25}\cdot 2^{-n}\right)\mathrm{u}[n-1]$$

d) $(D-1)(D-2)^2y[n]=3D^2x[n]\;/^{E^3}$

$(1-E)(1-2E)^2y[n]=3Ex[n]$

$$y_{us}[n]=\frac{3}{(1-E)(1-2E)^2}Ex[n]=\frac{3}{(1-E)(1-2E)^2}\left(7^{n+1}+3^{-n-1}\right)=-\frac{7^{n+1}}{338}+\frac{27\cdot 3^{-n}}{2}$$

6. zadatak (20)

a) Signal je očigledno paran pa ima samo parni deo

b+c)

$$v(t)=\frac{1}{5-3\cos\omega t}=\frac{2}{10-6\cos\omega t}=\frac{2}{1-2\cdot 3\cos\omega t+3^2}=\frac{2}{9}\frac{1}{\frac{1}{3^2}-2\cdot\frac{1}{3}\cos\omega t+1}$$

$$v(t) = \frac{2}{9}\left(\frac{1}{1-\frac{1}{3^2}}\right)\frac{1-\frac{1}{3^2}}{\frac{1}{3^2} - 2\cdot\frac{1}{3}\cos\omega t + 1} = \frac{1}{4}\,\frac{1-a^2}{a^2 - 2\cdot a\cos\omega t + 1}$$

$$v(t) = \frac{1}{4} + \frac{1}{2}\sum_{k=1}^{\infty}(1/3)^k \cos k\omega t \Rightarrow A[0] = 1/4,\ A[k] = 1/\left(2\cdot 3^k\right),\ B[k] = 0$$

$$X[0] = A[0],\ X[k] = A[k]/2$$

d) Razvija se samo aktivna snaga

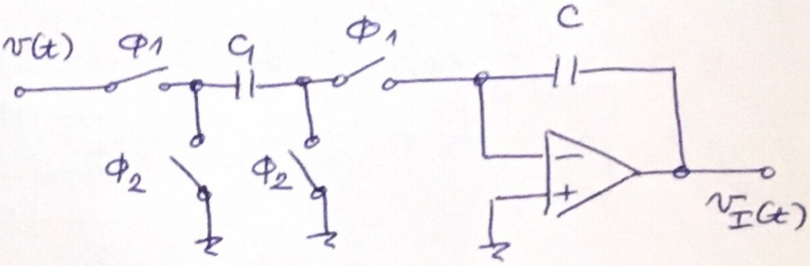
$$P = \frac{100\mathrm{V}^2}{2.5\Omega}\left(\frac{1}{16} + \frac{1}{8}\sum_{k=1}^{\infty}\left((1/3)^k\right)^2\right) = 40\mathrm{W}\left(\frac{1}{16} + \frac{1}{8}\sum_{k=1}^{\infty}\frac{1}{9^k}\right) =$$

$$= 40\mathrm{W}\left(\frac{1}{16} + \frac{1}{8}\cdot\frac{1/9}{1-1/9}\right) = 40\mathrm{W}\left(\frac{1}{16} + \frac{1}{64}\right) = \frac{25}{8}\mathrm{W} = 3.125\mathrm{W}$$

$\boxed{1}$ $\Phi_1: \; nT_S \leq t < (n+\frac{1}{2})T_S$

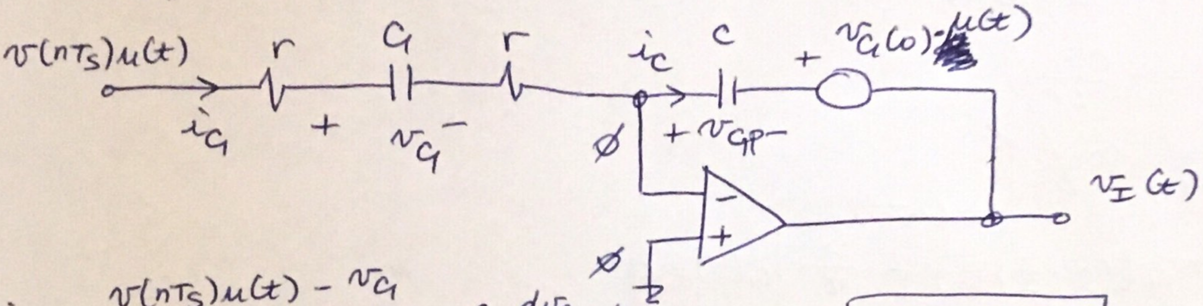
$\Phi_2: \; (n+\frac{1}{2})T_S \leq t < (n+1)T_S$

$C_1 = kC$ и $\tau = 2krC$



a) $v_{C_1}(nT_S) = 0$, $v_C(nT_S) = -v_I(nT_S)$

$v(nT_S)$



$$-i_{C_1} = \frac{v(nT_S)u(t) - v_{C_1}}{2r} = C_1 \frac{dv_{C_1}}{dt} \Big/ \mathcal{L}$$

$\boxed{\tilde{t} \equiv t - nT_S}$

$$\frac{v(nT_S)}{2r \cdot s \cdot C_1} = sV_{C_1}(s) - \cancel{v_{C_1}(0)}^{\,0} + \frac{1}{2rC_1} V_{C_1}(s)$$

$$(s + \frac{1}{\tau}) V_{C_1}(s) = \frac{v(nT_S)}{\tau \cdot s}$$

$$V_{C_1}(s) = \frac{v(nT_S)}{\tau \cdot s(s+1/\tau)} = \frac{v(nT_S)}{s} - \frac{v(nT_S)}{s+1/\tau}$$

$$v_{C_1}(\tilde{t}) = v(nT_S)(1 - e^{-\tilde{t}/\tau})\,u(\tilde{t})$$

$$-i_C = i_{C_1} \iff C\frac{dv_{CP}}{dt} = C_1 \cdot \frac{dv_{C_1}}{dt}$$

$$V_{CP}(s) = \frac{C_1}{C} \cdot V_{C_1}(s) = k \cdot V_{C_1}(s)$$

$$V_I(s) = -V_C(s) - \frac{v_C(nT_S)}{s} = -k \cdot \left(\frac{v(nT_S)}{s} - \frac{v(nT_S)}{s+1/\tau}\right) + \frac{v_I(nT_S)}{s}$$

$$v_I(\tilde{t}) = -k \cdot v(nT_S)(1 - e^{-\tilde{t}/\tau})\,u(\tilde{t}) + v_I(nT_S)\,u(\tilde{t})$$

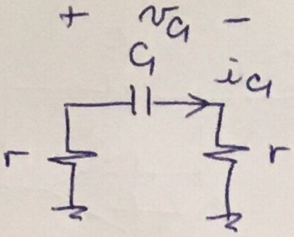
$\tilde{t}_1 = \frac{T_S}{2}$ односно за: $t_1 = (n+\frac{1}{2})T_S$ je:

$$v_{C_1}(t_1) \cong \lim_{\tilde{t}\to\infty} v_{C_1}(\tilde{t}) = v(nT_S)$$

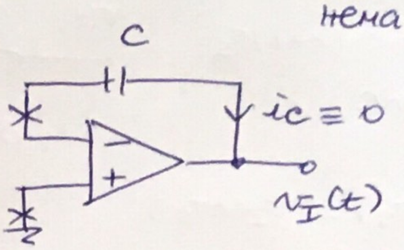
$$v_I(t_1) \cong \lim_{\tilde{t}\to\infty} v_I(\tilde{t}) = -k\,v(nT_S) + v_I(nT_S)$$

$$v_C(t_1) \cong \lim_{\tilde{t}\to\infty} v_C(\tilde{t}) = k \cdot v(nT_S) - v_I(nT_S)$$

в)



нема струје



$$v_I(t) = v_I\left(\left(n+\tfrac{1}{2}\right)T_s\right) = \text{const} \quad ; \quad v_C(t) \equiv v_C\left(\left(n+\tfrac{1}{2}\right)T_s\right)$$

$$i_{C1} = C_1 \frac{dv_{C1}}{dt} = -\frac{v_{C1}}{2r}$$

$$\frac{dv_{C1}}{dt} + \frac{v_{C1}}{2rC_1} = 0 \ \Big/ \ \mathcal{L}$$

$$sV_{C1}(s) - v_{C1}\underbrace{\cancel{\left(\left(n+\tfrac{1}{2}\right)T_s\right)}}_{v(nT_s)} + \frac{1}{\tau}V_{C1}(s) = 0$$

$$V_{C1}(s) = \frac{v(nT_s)}{s + 1/\tau}$$

$$v_{C1}(\tilde{t}) = v(nT_s)\left(1 - e^{-\tilde{t}/\tau}\right)u(\tilde{t}) \qquad \tilde{t} = t - \left(n+\tfrac{1}{2}\right)T_s$$

$$v_{C1}((n+1)T_s) = \lim_{\tilde{t}\to\infty} v_{C1}(\tilde{t}) = 0$$

$$v_I((n+1)T_s) = \lim_{\tilde{t}\to\infty} v_I(\tilde{t}) = -k\cdot v(nT_s) + v_I(nT_s)$$

$$v_C((n+1)T_s) = \lim_{\tilde{t}\to\infty} v_C(\tilde{t}) = kv(nT_s) - v_I(nT_s)$$

c) Увек је на почетку периоде: $v_{C1}(nT_s) = 0$, $v_C(nT_s) = -v_I(nT_s)$;

а на крају је $\boxed{v_I((n+1)T_s) = -kv(nT_s) + v_I(nT_s)}$; $v_C((n+1)T_s) = 0$ и

$v_C((n+1)T_s) = -v_I((n+1)T_s)$.

Ако се заокружена једначина напише као диференцна, добија се:

$$v_I[n+1] - v_I[n] = -kv[n]$$