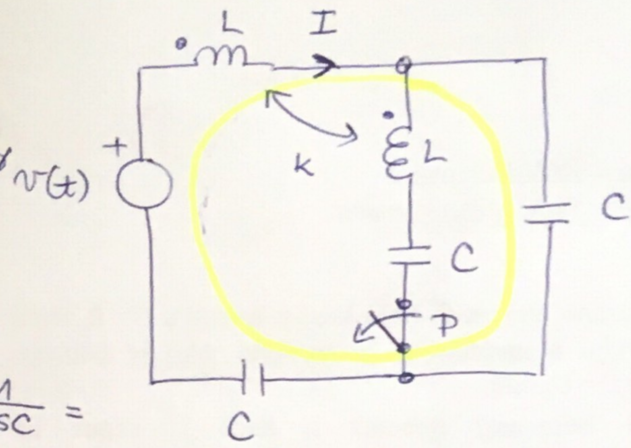
КОЛОКВИЈУМ 2

1) $v(t) = U_0 u(t)$

a)

$$V(s) - sL\,I(s) - \frac{2}{sC} I(s) = 0$$



$$V_C(s) = \frac{1}{sC} I(s)$$

$$\Rightarrow V_C(s) = \frac{V(s)}{sL + \frac{2}{sC}} \cdot \frac{1}{sC} =$$

$$= \frac{V(s)}{s^2LC + 2} = \frac{U_0}{s(s^2LC+2)} =$$

$$s_{1,2} = \pm j\sqrt{\frac{2}{LC}}$$

$$A = \lim_{s \to 0} \cancel{s} \cdot \frac{U_0}{\cancel{s}(s^2LC+2)} = \frac{U_0}{2}$$

$$Q = \frac{1}{\beta} \lim_{s \to a} ((s-\alpha)^2 + \beta^2) V_C(s) = \sqrt{\frac{LC}{2}} \lim_{s \to j\sqrt{\frac{2}{LC}}} \cancel{\left(s^2 + \frac{2}{LC}\right)} \cdot \frac{U_0}{s\cancel{(s^2 LC + 2)}_{LC}} =$$

$$= \frac{U_0}{\sqrt{2LC}} \lim_{s \to j\sqrt{\frac{2}{LC}}} \frac{1}{s} = \frac{U_0}{\sqrt{2\cancel{LC}}} \cdot \frac{1}{j\sqrt{\frac{2}{\cancel{LC}}}} = -j\frac{U_0}{2}$$

$$v_C(t) = \frac{U_0}{2} u(t) - \frac{U_0}{2} \cos\left(\overset{(\omega_0 t \cdot \sqrt{2})}{\sqrt{\frac{2}{LC}}\, t}\right) \cdot u(t)$$

$v_C(t) = max$ за $\cos(\omega_0 t_1) = min$

$$\sqrt{\frac{2}{LC}}\, t_1 = \pi$$

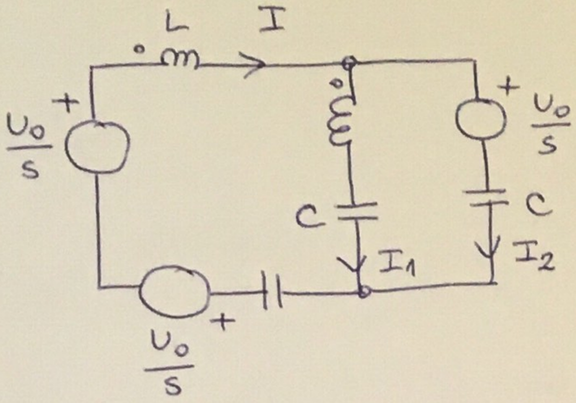
$$t_1 = \frac{\pi\sqrt{LC}}{\sqrt{2}} = \frac{\sqrt{2LC}\,\pi}{2} = \frac{\pi\sqrt{2}}{2\omega_0}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow LC = \frac{1}{\omega_0^2}$$

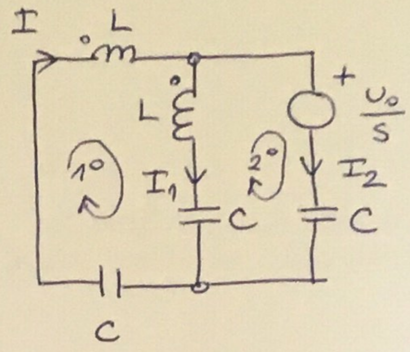
в) $v_C(t_1^-) = U_0$

$$i_C(t) = C\frac{dv_C}{dt} = C \cdot \frac{U_0}{2}\left(\delta(t) - \cos(\omega_0\sqrt{2} \cdot t) \cdot \delta(t) + \omega_0\sqrt{2} \cdot \sin(\omega_0\sqrt{2}\, t)\, u(t)\right)$$

$$i_C(t_1) = \frac{CU_0}{2} \cdot \omega_0\sqrt{2} \cdot \sin\left(\cancel{\omega_0\sqrt{2}} \cdot \frac{\pi}{\cancel{\omega_0\sqrt{2}}}\right) \cdot \overset{1}{\cancel{u(t_1)}} = 0$$



$\Leftrightarrow$



$1°$ $$-sLI - skLI_1 - sLI_1 - skLI - \frac{1}{sC}I_1 - \frac{1}{sC}I = 0$$

$$I_1 \cdot \left(sL + \cancel{skL} + \frac{1}{sC}\right) = -\left(sL + \cancel{skL} + \frac{1}{sC}\right)I$$

$$I_1 = -I$$

$$I_2 = I - I_1 = 2I$$

$2°$ $$\frac{1}{sC}I_1 + sLI_1 + skLI = \frac{1}{sC}I_2 + \frac{U_0}{s}$$

$$-\frac{1}{sC}I \underbrace{- \cancel{sLI} + \frac{1}{2}\cancel{sLI_1}}_{-\frac{1}{2}sLI} = \frac{2I}{sC} + \frac{U_0}{s}$$

$$-I\left(\frac{3}{sC} + \frac{1}{2}sL\right) = \frac{U_0}{s} \quad / \cdot 2sC$$

$$-I(6 + s^2LC) = 2U_0C$$

$$I(s) = -\frac{2U_0C}{\frac{s^2}{\omega_0^2} + 6} = -\frac{2U_0C \cdot \omega_0^2}{s^2 + 6\omega_0^2} = -2U_0C\,\omega_0^{\cancel{2}} \cdot \frac{1}{\cancel{\omega_0}\sqrt{6}} \cdot \underbrace{\frac{\omega_0\sqrt{6}}{s^2 + (\omega_0\sqrt{6})^2}}_{\mathcal{L}\{\sin\}}$$

$$i(\tilde{t}) = -\cancel{2}\omega_0C \cdot U_0 \cdot \frac{\sqrt{6}}{\cancel{6}_{3}} \cdot \sin(\omega_0\sqrt{6}\,\tilde{t})\,u(\tilde{t})$$

$\tilde{t}$ – померено за t_1 од 0

$$\Rightarrow i(t) = -\omega_0C\sqrt{6} \cdot \frac{U_0}{3} \sin(\omega_0\sqrt{6} \cdot (t - t_1))\,u(t - t_1)$$

3. zadatak

Data je Z transformacija prenosne funkcije sistema $h[n]$:

$H(z) = (2z^2 - 0.75z)/\left((z-0.25)(z-0.5)\right)$.

a) [5] ROC: $|z| > 0.5$; $h[n] = \left(0.25^n + 0.5^n\right)u[n]$, stabilno

b) [5] ROC: $|z| < 0.25$; $h[n] = \left(-0.25^n - 0.5^n\right)u[-n-1]$, nestabilno

c) [5] ROC: $0.25 < |z| < 0.5$; $h[n] = 0.25^n u[n] - 0.5^n u[-n-1]$, nestabilno

d) [5] $D\Delta^2$ u[n]= $D\Delta$ (u[n+1] –u[n])= $D\Delta$ $\delta[n+1]$= Δ $\delta[n]$= $\delta[n+1]$– $\delta[n]$;

$y[n] = h[n+1] - h[n]$, u sva tri slučaja.

4. zadatak

a) $H(z) = \dfrac{3z}{(3z-1)^2} + \dfrac{3z}{3z-1} = \dfrac{9}{9-6z^{-1}+z^{-2}}$

$$(9-6z^{-1}+z^{-2})Y(z) = 9X(z)$$

$$9y[n] - 6y[n-1] + y[n-2] = 9x[n]$$

b) Videti vežbe.

c) Sopstveni odziv se dobija na sledeći način.

$$9(z^2Y(z) - z^2y[0] - zy[1]) - 6(zY(z) - zy[0]) + Y(z) = z^2X(z) - z^2x[0] - zx[1]$$

$$Y(z) = \frac{z^2}{9z^2+6z+1}X(z) - \frac{\frac{18}{5}z^2 + \frac{108}{5}z}{9z^2-6z+1} = Y_P(z) + Y_S(z)$$

$$Y_S(z) = -\frac{\frac{18}{5}z^2 + \frac{108}{5}z}{9z^2-6z+1} = -\frac{2z^2}{5(z-1/3)^2} - \frac{12z}{5(z-1/3)^2}$$

$$y_S[n] = -\left(\frac{12}{5} + \frac{2}{5}(n+1)\right)3^{-n}$$

Prinudni odziv je:

$$Y_P(z) = X(z)H(z) = \frac{z^3}{(z-5)(z-1/3)^2} = \frac{Az}{z-5} + \frac{Bz}{z-1/3} + \frac{Cz}{(z-1/3)^2}$$

$$A = \frac{225}{196},\ B = -\frac{29}{196},\ C = -\frac{1}{42}$$

$$y_P[n] = -\frac{29}{196}3^{-n}\,\mathrm{u}[n] - \frac{1}{14}3^{-n}n\mathrm{u}[n] + \frac{225}{196}5^n\,\mathrm{u}[n]$$