1

$$\nabla(\Delta-2)^2 y[n] = Dx[n-1]$$

$$E\nabla(\Delta-2)^2 y[n] = ED\, x[n-1]$$

$$\Delta x[n] = x[n+1] - x[n] = (E-1)\, x[n]$$

$$E\nabla x[n] = \Delta x[n]$$

$$(E-1)(E-3)^2 y[n] = x[n-1] \Rightarrow (E-1)(E^2-6E+9)\, y[n] = x[n-1]$$

$$(E^3-7E^2+15E-9)\, y[n] = x[n-1]$$

a)

$$h[n+3] - 7h[n+2] + 15h[n+1] - 9h[n] = \delta[n-1] \;/\cdot D^3$$

$$h_1[n] - 7h_1[n-1] + 15h_1[n-2] - 9h_1[n-3] = \delta[n]$$

$$h_1[0] = 1$$

$$h_1[1] = 7h_1[0] = 7$$

$$h_1[2] = 7h_1[1] - 15h_1[0] = 49 - 15 = 34$$

$$P(\lambda) = \lambda^3 - 7\lambda^2 + 15\lambda - 9 = (\lambda-1)(\lambda-3)^2$$

$$\lambda_1 = 1, \quad \lambda_{2,3} = 3$$

$$h_1[n] = C_1 1^n u[n] + 3^n(C_2 + nC_3)\, u[n]$$

$$h_1[0] = C_1 + C_2 = 1$$

$$h_1[1] = C_1 + 3C_2 + 3C_3 = 7$$

$$h_1[2] = C_1 + 9C_2 + 18C_3 = 34$$

$$2C_2 + 3C_3 = 6 \;/\cdot 4$$

$$8C_2 + 18C_3 = 33$$

$$6C_3 = 9 \Rightarrow C_3 = \frac{3}{2}$$

$$C_2 = \frac{6 - 3\cdot\frac{3}{2}}{2} = \frac{3}{4}$$

$$C_1 = \frac{1}{4}$$

$$h_1[n] = \left(\frac{1}{4} + \left(\frac{3}{4} + \frac{3}{2}n\right)3^n\right)u[n]$$

Систем је нестабилан.

$$h[n] = \left(\frac{1}{4} + \left(\frac{3}{4} + \frac{3}{2}(n-4)\right)\cdot 3^{n-4}\right)u[n-4]$$

б) $y[0] = y[2] = 0, \; y[1] = 1$

$$y_s[n] - 7y_s[n-1] + 15y_s[n-2] - 9y_s[n-3] = 0$$

$$y_s[n] = (C_1 + (C_2 + nC_3)\cdot 3^n)\, u[n]$$

$$y_s[0] = C_1 + C_2 = 0$$

$$y_s[1] = C_1 + 3C_2 + 3C_3 = 1$$

$$y_s[2] = C_1 + 9C_2 + 18C_3 = 0$$

$$\Rightarrow y_s[n] = -\frac{3}{2} + \left(\frac{3}{2} - \frac{2}{3}n\right)\cdot 3^n$$

$$2C_2 + 3C_3 = 1$$

$$8C_2 + 18C_3 = 0$$

$$6C_3 = -4, \quad C_3 = -\frac{2}{3}$$

$$C_2 = \frac{1 + \cancel{3}\cdot\frac{2}{\cancel{3}}}{2} = \frac{3}{2}$$

$$C_1 = -\frac{3}{2}$$

c) $P(E) = E^3 - 7E^2 + 15E - 9$

$P(E)\, y_{pa}[n] = x[n-1] = x_1[n-1] + x_2[n-1]$

$y_{pa1}[n] = \frac{1}{P(E)} 2^{-(n-1)} = 2^{-(n-1)} \frac{1}{P(1/2)} = -\frac{8}{25} 2^{-(n-1)}$

$y_{pa2}[n] = \frac{\sin(\pi(n-1))}{P(-1)} = -\frac{\sin((n-1)\pi)}{32} \equiv \emptyset$

$y_{pa}[n] = -\frac{16}{25} \cdot 2^{-n} u[n]$

$y_p[n] = y_h[n] + y_{pa}[n]$

$y_p[0,1,2,3] = \emptyset$ jер je $y_p[n] - 7y_p[n-1] + 15y_p[n-2] - 9y_p[n-3] = x[n-4]$

$y_h[0] = y_p[0] - y_{pa}[0] = \frac{16}{25}$

$y_h[1] = \frac{8}{25}$

$y_h[2] = \frac{4}{25}$

$y_h[n] = (C_1 + (C_2 + nC_3) \cdot 3^n) u[n]$

$C_1 + C_2 + 0 \cdot C_3 = \frac{16}{25}$

$C_1 + 3C_2 + 3C_3 = \frac{8}{25}$

$C_1 + 9C_2 + 18C_3 = \frac{4}{25}$

$2C_2 + 3C_3 = -\frac{8}{25}$

$8C_2 + 18C_3 = -\frac{12}{25}$

$6C_3 = \frac{12}{25}$; $C_3 = \frac{2}{25}$

$C_2 = -\frac{7}{25}$

$C_1 = \frac{23}{25}$

$y_p[n] = \left(-\frac{16}{25} 2^{-n} + C_1 + (C_2 + nC_3) 3^n\right) u[n]$

2. zadatak (30)
Kontinualni LTI system je opisan diferencijalnom jednačinom: $(D-1)(D-2)y(t)=(D+1)x(t)$.
a) [10] odrediti impulsni odziv sistema.
b) [20] Odrediti prinudni odziv sistema ako je $x(t)=\left(1+\cos 2t\right)\cdot \mathrm{u}(t)$

Resenje:
a)

$$(D-1)(D-2)s_1(t)=\mathrm{u}(t)$$

$$s_1(t)=c_1e^t+c_2e^{2t}+\left.\frac{1}{(D-1)(D-2)}\right|_{D=0}=c_1e^t+c_2e^{2t}+\frac{1}{2}$$

$$\frac{d}{dt}s_1(t)=c_1e^t+2c_2e^{2t}$$

$$\left.\begin{aligned}c_1+c_2&=-1/2\\ c_1+2c_2&=0\end{aligned}\right\}c_2=1/2, c_1=-1$$

$$s(t)=s_1(t)+\frac{d}{dt}s_1(t)=2c_1e^t+3c_2e^{2t}+1/2$$

$$h(t)=\left(\frac{d}{dt}s(t)\right)\mathrm{u}(t)=\left(2c_1e^t+6c_2e^{2t}\right)\mathrm{u}(t)=\left(-2e^t+3e^{2t}\right)\mathrm{u}(t)$$

b)

$$\frac{1}{(D-1)(D-2)}=-\frac{1}{D-1}+\frac{1}{D-2}=-\frac{D+1}{D^2-1}+\frac{D+2}{D^2-4}$$

$$-\frac{D+1}{-\omega^2-1}+\frac{D+2}{-\omega^2-4}=\frac{D+1}{5}-\frac{D+2}{8}=\frac{3}{40}D-\frac{2}{40}$$

$$y_1(t)=c_1e^t+c_2e^{2t}+\frac{1}{2}+\left(\frac{3}{40}D-\frac{1}{20}\right)\cos 2t$$

$$y_1(t)=c_1e^t+c_2e^{2t}+\frac{1}{2}-\frac{3}{20}\sin 2t-\frac{1}{20}\cos 2t$$

$$\frac{d}{dt}y_1(t)=c_1e^t+2c_2e^{2t}-\frac{6}{20}\cos 2t+\frac{2}{20}\sin 2t$$

$$\left.\begin{aligned}c_1+c_2&=-1/2+1/20\\ c_1+2c_2&=6/20\end{aligned}\right\}c_2=15/20, c_1=-24/20$$

$$y(t)=\left(y_1(t)+\frac{d}{dt}y_1(t)\right)\mathrm{u}(t)=\left(-\frac{48}{20}e^t+\frac{45}{20}e^{2t}+\frac{1}{2}-\frac{1}{20}\sin 2t-\frac{7}{20}\cos 2t\right)\mathrm{u}(t)$$

3. zadatak (10)
Dat je signal čija je osnovna perioda definisana sa

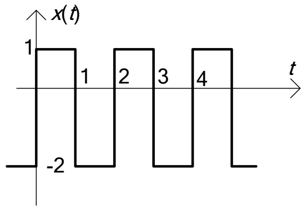
$$x_F(t)=\begin{cases}1, 0\le t\le 1\\ -2, 1<t<2\end{cases}$$

a) [5] ako je $g(t)=\sum_{k=-\infty}^{\infty}\delta(t-2k)$, pokazati da je $y(t)=\frac{d}{dt}x(t)=Ag(t-t_1)+Bg(t-t_2)$ i naći A,t_1,B,t_2

b) [5] Naći razvoj u kompleksni Furijov red signala $y(t)$ na njegovoj osnovnoj periodi.

Resenje:

a)



$$y(t)=\sum_{k=-\infty}^{\infty}3\delta(t-2k)-\sum_{k=-\infty}^{\infty}3\delta(t-2k-1)\Rightarrow A=B=3,t_1=0,t_2=1$$

b)

$$T_F=2,\omega_F=\pi$$

$$y_F(t)=3\delta(t)-3\delta(t-1)\Rightarrow Y(j\omega)=3-3e^{-j\omega}\Rightarrow Y[k]=\frac{1}{2}\left(3-3e^{-jk\pi}\right)$$

4. zadatak (10)

Neka je konvolucijom dobijen signal $g(t)=\left(x(t)\cos^2 t\right)*\frac{\sin t}{\pi t}$. Ako je $x(t)$ realan signal takav da je $H(j\omega)=0,$ za $|\omega|\geq 1$, naći $h(t)$ LTI sistema koji na pobudu $x(t)$ daje odziv $g(t)$.

Resenje

$$g(t)=\left(x(t)\cos^2 t\right)*\frac{\sin t}{\pi t}=\frac{1}{2}x(t)(1+\cos 2t)*\frac{\sin t}{\pi t}=\frac{1}{2}x(t)*\frac{\sin t}{\pi t}+\frac{1}{2}\left(\underbrace{x(t)\cdot\cos 2t}_{\text{modulacija}}\right)*\frac{\sin t}{\pi t}$$

$$G(j\omega)=\frac{1}{2\pi}X(j\omega)\cdot\mathrm{rect}(\omega/2)+\frac{1}{4}\left(X(j(\omega-2))+X(j(\omega-2)\right)\mathrm{rect}(\omega/2)$$

Kako je

$$G(j\omega)=X(j\omega)\cdot H(j\omega)$$

znači da je

$$H(j\omega)=G(j\omega)/X(j\omega)=\frac{1}{2\pi}\mathrm{rect}(\omega/2)+\frac{1}{4}\frac{\left(X(j(\omega-2))+X(j(\omega-2)\right)\mathrm{rect}(\omega/2)}{X(j\omega)}$$

Prethodni izraz ne sme da zavisi od $X(j\omega)$ što je moguce samo kad je drugi sabirak jednak nuli, odnosno

$$\left(X(j(\omega-2))+X(j(\omega-2)\right)\mathrm{rect}(\omega/2)=0$$

iz čega proizilazi da je i $X(j\omega)=0,$ za $|\omega|\geq 1$

pa je $H(j\omega)=\frac{1}{2\pi}\mathrm{rect}(\omega/2),\Rightarrow h(t)=\frac{1}{2}\frac{\sin t}{\pi t}$

5. zadatak (30)

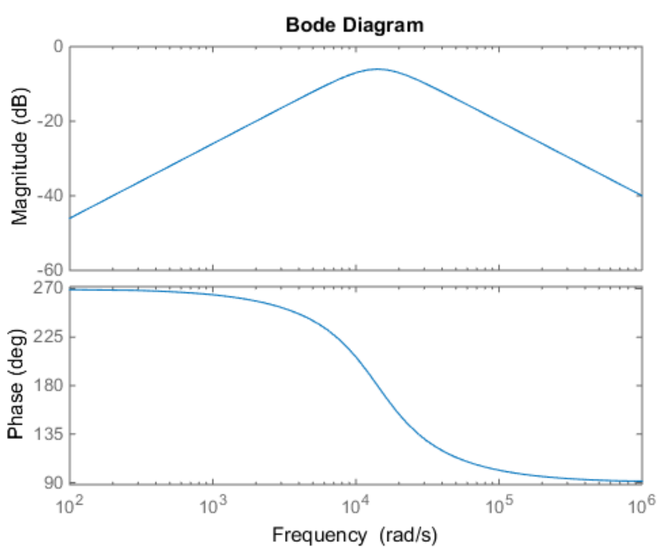
Rešenje:

a) Prenosna funkcija je filtar propusnik opsega učestanosti:

$$H(s)=\frac{-\dfrac{s}{C_2R_1}}{s^2+s\dfrac{C_1+C_2}{C_1C_2R_2}+\dfrac{R_1+R_0}{C_1C_2R_2R_1R_0}}=\frac{-\dfrac{s}{CR}}{s^2+s\dfrac{2}{CR}+\dfrac{2}{(CR)^2}}$$

Očigledno je $\omega_P=\dfrac{\sqrt{2}}{CR}$ i $Q=\dfrac{\sqrt{2}}{2}$.

b) Bodeovi dijagrami su prikazani na slici.



c) Prenosna funkcija se može napisati kao:

$$H(s)=-\frac{s\omega_0}{(s+a)^2+\omega_0^2},$$

gde je $a=\omega_0=10000\dfrac{\text{rad}}{\text{s}}$. Kako je $V_U(s)=A\dfrac{a+s}{(a+s)^2+\omega_0^2}$ za $A=10\ \text{mV}$, dobija se

$$V_I(s) = -A\frac{\omega_0 s(s+a)}{((a+s)^2+\omega_0^2)^2}\,.$$

Na osnovu teoreme je $\mathcal{F}\{t\sin(\omega_0 t)\mathrm{e}^{-at}\,\mathrm{u}(t)\} = \frac{2\omega_0(s+a)}{((a+s)^2+\omega_0^2)^2}$. Kako je $\mathcal{F}\left\{\frac{dx(t)}{dt}\right\} = sX(s)$, dobija se da je

$$V_I(s) = -\frac{sA}{2}\cdot\mathcal{F}\{t\sin(\omega_0 t)\mathrm{e}^{-at}\,\mathrm{u}(t)\}\,,$$

pa je $v_I(t) = -\frac{A}{2}\frac{d}{dt}\left(t\sin(\omega_0 t)\mathrm{e}^{-at}\,\mathrm{u}(t)\right) = -\frac{A}{2}((1-at)\sin(\omega_0 t)+\omega_0 t\cos(\omega_0 t))\mathrm{e}^{-at}\,\mathrm{u}(t)$.

d) Kako je ovo propusnik opsega učestanosti, jako je veliko slabljenje za $\omega = 0$ i $\omega = 10^6$. Dobija se izlazni napon isto kao pod c) sa kašnjenjem. Ova komponenta iščezava sa vremenom, tako da je u ustaljenom stanju napon na izlazu jednak nuli.