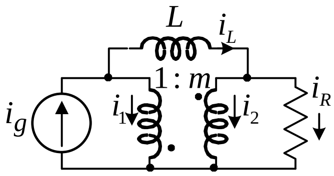
Signali i sistemi – Rešenja zadataka sa ispita od 07.02.2016.

1. zadatak (30 poena)



$$i_G = i_L + i_1 = i_L + m i_2$$

$$i_L = i_2 + i_R = \frac{1}{m}(i_G - i_L) + i_R \Rightarrow i_R = \left(\frac{m+1}{m}\right) i_L - \frac{1}{m} i_G$$

$$R i_R = -L\frac{di_L}{dt} - \frac{1}{m} R i_R \Rightarrow \frac{R}{L}\left(\frac{m+1}{m}\right) i_R = -\frac{di_L}{dt}$$

$$\frac{R}{L}\left(\frac{m+1}{m}\right)^2 i_L - \frac{R}{L}\left(\frac{m+1}{m^2}\right) i_G = -\frac{di_L}{dt}$$

$$\frac{di_L}{dt} + A \cdot i_L = B \cdot i_G$$

$$sI_L - I_0 + A \cdot I_L = B \cdot I_m \frac{1}{s} + B\phi e^{-sT}$$

$$I_L = \frac{I_0}{s+A} + \frac{BI_m}{s(s+A)} + \frac{B\phi}{s+A} = \frac{I_0}{s+A} + \frac{BI_m}{As} - \frac{BI_m}{A(s+A)} + \frac{B\phi}{s+A} e^{-sT}$$

$$i_L(t) = \underbrace{I_0 e^{-At}\mathrm{u}(t)}_{sopstveni} + \underbrace{\overbrace{\frac{I_m}{m+1}\mathrm{u}(t)}^{ustaljeni} - \frac{I_m}{m+1} e^{-At}\mathrm{u}(t) + B\phi e^{-A(t-T)}\mathrm{u}(t-T)}_{prinudni}$$

2. zadatak (30 poena)

$$3zH(z) - 2H(z) - z^{-1}H(z) = 3 \Rightarrow H = \frac{3z}{3z^2 - 2z - 1} = \frac{3}{4}\frac{z}{z-1} z^{-1} + \frac{1}{4}\frac{z}{z+1/3} z^{-1}$$

$$h[n] = \frac{3}{4}\mathrm{u}[n-1] + \frac{1}{4}\left(-1/3\right)^{n-1}\mathrm{u}[n-1]$$

b)

$$\mathrm{u}[n] = 0 \text{ za } n < 0 \Rightarrow \frac{n \cdot \mathrm{u}[n] \cdot \pi}{3} = 0 \text{ za } n < 0 \Rightarrow \sin\left(\frac{n \cdot \mathrm{u}[n] \cdot \pi}{3}\right) = 0 \text{ za } n < 0$$

$$\mathrm{u}[n] = 1 \text{ za } n \geq 0 \Rightarrow \frac{n \cdot \mathrm{u}[n] \cdot \pi}{3} = \frac{n \cdot \pi}{3} \text{ za } n \geq 0 \Rightarrow \sin\left(\frac{n \cdot \mathrm{u}[n] \cdot \pi}{3}\right) = \sin\left(\frac{n \cdot \pi}{3}\right)\mathrm{u}[n]$$

$$n \cdot 4^n \mathrm{u}[-n] = 0 \text{ za } n \geq 0 \Rightarrow \left(3^{-n} + n \cdot 4^n \mathrm{u}[-n]\right)\sin\left(\frac{n \cdot \pi}{3}\right)\mathrm{u}[n] = 3^{-n}\sin\left(\frac{n \cdot \pi}{3}\right)\mathrm{u}[n]$$

$$\Rightarrow x[n] = 3^{-n}\sin\left(\frac{n \cdot \pi}{3}\right)\mathrm{u}[n]$$

$$\left.\begin{array}{l}\left(3y[2]-2y[1]-y[0]=3x[1]=\sqrt{3}/2\right)\times 2\\ \left(3y[3]-2y[2]-y[1]=3x[2]=\sqrt{3}/6\right)\times 3\end{array}\right\}\left.\begin{array}{c}6y[2]-4y[1]=\sqrt{3}\\ 9\sqrt{3}-6y[2]-3y[1]=\sqrt{3}/2\end{array}\right\} y[1]=\frac{15}{14}\sqrt{3}$$

$$3y[n+2]-2y[n+1]-y[n]=3x[n+1]$$

$$3\left(z^2Y(z)-z^2\underbrace{y[0]}_{0}-zy[1]\right)-2\left(zY(z)-z\underbrace{y[0]}_{0}\right)-Y(z)=3(zX(z)-z\underbrace{x[0]}_{0})$$

$$3\left(z^2Y(z)-zy[1]\right)-2zY(z)-Y(z)=3z\frac{\frac{1}{3}z\sin(\pi/3)}{z^2-\frac{2}{3}z\cos(\pi/3)+\frac{1}{9}}$$

$$Y(z)=\frac{3z}{3z^2-2z-1}\frac{\frac{1}{3}z\sin(\pi/3)}{z^2-\frac{2}{3}z\cos(\pi/3)+\frac{1}{9}}+\frac{3z}{3z^2-2z-1}y[1]=$$

$$=\frac{z}{(z-1)(z+1/3)}\cdot\frac{z\sqrt{3}/6}{z^2-z/3+1/9}+\frac{z}{(z-1)(z+1/3)}y[1]=$$

$$\frac{C_1}{z-1}+\frac{C_2}{z+1/3}+\frac{Az+B}{(z-1/6)^2+(1/2\sqrt{3})^2}$$

$$C_1=\frac{1}{(1+1/3)}\cdot\frac{1\sqrt{3}/6}{1-1/3+1/9}+\frac{1}{(1+1/3)}\frac{15}{14}\sqrt{3}=\frac{27}{28}\sqrt{3}$$

$$C_2=\frac{-1/3}{(-1/3-1)}\cdot\frac{(-1/3)\sqrt{3}/6}{(-1/3)^2-(-1/3)/3+1/9}+\frac{(-1/3)}{(-1/3-1)}\frac{15}{14}\sqrt{3}=\frac{19}{28}\frac{\sqrt{3}}{3}$$

$$Q=\sqrt{12}\frac{(1/6+j/\sqrt{12})^2}{(1/6+j/\sqrt{12}-1)(1/6+j/\sqrt{12}+1/3)}\sqrt{3}/6=\frac{j+\sqrt{3}}{9j-\sqrt{3}}$$

$$A=\mathrm{Im}\{Q\}=-\frac{5}{14}\frac{\sqrt{3}}{3},\ \ \mathrm{Re}\{Q\}=1/14,\ \ B=\frac{1}{2\sqrt{3}}\mathrm{Re}\{Q\}+A\cdot\frac{1}{6}=\frac{1}{14}\left(\frac{\sqrt{3}}{6}+\frac{5}{6}\frac{\sqrt{3}}{3}\right)=\frac{2}{21}\frac{\sqrt{3}}{3}$$

$$\frac{Az+zBz^{-1}}{z^2-\frac{2}{3}z\cos(\pi/3)+\frac{1}{9}}=\frac{-\frac{5}{7}3^{-1}z\cdot\sin(\pi/3)+\frac{4}{21}3^{-1}z\cdot\sin(\pi/3)z^{-1}}{z^2-\frac{2}{3}z\cos(\pi/3)+\frac{1}{9}}$$

$$y[n]=\frac{27}{28}\sqrt{3}\cdot\mathrm{u}[n-1]+\frac{19}{28}\frac{\sqrt{3}}{3}(-3)^{-n+1}\mathrm{u}[n-1]$$
$$-\frac{5}{7}3^{-n}\sin(n\pi/3)\mathrm{u}[n]+\frac{4}{21}3^{-n+1}\sin((n-1)\pi/3)\mathrm{u}[n-1]$$

3. zadatak (15 poena)

Prema osobini skaliranja vremenske ose (relacija 6.28 iz udžbenika) imamo da je: $x(3t)\xleftrightarrow{F}\frac{1}{3}X(j\frac{\omega}{3})$, $h(3t)\xleftrightarrow{F}\frac{1}{3}H(j\frac{\omega}{3})$, $g(3t)\xleftrightarrow{F}\frac{1}{3}G(j\frac{\omega}{3})$ i $y(3t)\xleftrightarrow{F}\frac{1}{3}Y(j\frac{\omega}{3})$.

Takođe, na osnovu osobine konvolucije (relacija 6.32) imamo: $Y(j\omega) = X(j\omega)H(j\omega)$ i $\frac{1}{3}G(j\frac{\omega}{3}) = \frac{1}{3}X(j\frac{\omega}{3})\frac{1}{3}H(j\frac{\omega}{3}) = \frac{1}{9}X(j\frac{\omega}{3})H(j\frac{\omega}{3}) \Rightarrow G(j\frac{\omega}{3}) = \frac{1}{3}X(j\frac{\omega}{3})H(j\frac{\omega}{3})$.

Kako je: $Y(j\frac{\omega}{3}) = X(j\frac{\omega}{3})H(j\frac{\omega}{3})$, sledi da je $G(j\frac{\omega}{3}) = \frac{1}{3}Y(j\frac{\omega}{3})$, ili $G(j\omega) = \frac{1}{3}Y(j\omega)$.

Primenom inverzne Furijeove transformacije dobija se $g(t) = \frac{1}{3}y(t)$, odakle sledi:

$$A = \frac{1}{3},\ B = 1.$$

4. zadatak (15 poena)

Imamo da je $X(j\Omega) = \dfrac{1}{1-\frac{3}{4}e^{-j\Omega}}$ i $H(j\Omega) = \dfrac{1}{1-\frac{1}{2}e^{-j\Omega}}$, pa je odziv sistema u frekvencijskom domenu:

$$Y(j\Omega) = X(j\Omega)H(j\Omega) = \frac{1}{1-\frac{3}{4}e^{-j\Omega}}\frac{1}{1-\frac{1}{2}e^{-j\Omega}} = \frac{3}{1-\frac{3}{4}e^{-j\Omega}} + \frac{-2}{1-\frac{1}{2}e^{-j\Omega}},$$

a u vremenskom domenu:

$$y[n] = 3\left(\frac{3}{4}\right)^n u[n] - 2\left(\frac{1}{2}\right)^n u[n]$$

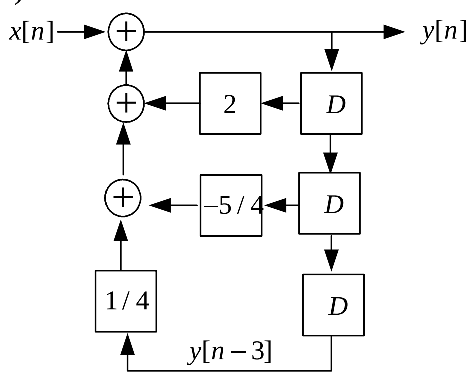
5. zadatak (10 poena)

Signal $x(t) = \left(\dfrac{\sin 5000\pi t}{\pi t}\right)$ ima pravougaoni spektar čija je maksimalna učestanost $\omega_{\max} = 5000\pi$. Dakle, minimalna potrebna učestanost odabiranja (Nikvistova brzina) za taj signal je $\omega_N = 2\omega_{\max} = 10000\pi$.

Spektar signal $g(t) = \left(\dfrac{\sin 5000\pi t}{\pi t}\right)^2$ predstavlja konvoluciju spektara signala $x(t)$. Taj spektar je trougaonog oblika, a maksimalna učestanost u spektru je $\omega_{\max} = 10000\pi$. Minimalna potrebna učestanost odabiranja (Nikvistova brzina) za taj signal je $\omega_N = 2\omega_{\max} = 20000\pi$.

6. zadatak (30 poena)

a)



b) $P(\lambda) = \lambda^2 - 2\lambda^2 + 1.25\lambda - 0.25 = (\lambda - 1)(\lambda - 1/2)^2 \Rightarrow y_S[n] = C_1 + (C_2 + nC_3)\frac{1}{2^n}$

$$\left.\begin{array}{l} y_s[0]=2=C_1+C_2 \\ y_s[1]=0=C_1+\frac{1}{2}C_2+\frac{1}{2}C_3 \\ y_s[2]=2=C_1+\frac{1}{4}C_2+\frac{1}{2}C_3 \end{array}\right\} C_1=10, C_2=-8, C_3=-12$$

c) $y_h[n]=C_1+(C_2+nC_3)\frac{1}{2^n}$

$$y_p[n-3]=\frac{1}{P(E)}(1/3)^n=\frac{1}{P(1/3)}(1/3)^n=\frac{1}{27}\frac{1}{P(1/3)}(1/3)^{n-3}=-2\cdot(1/3)^{n-3},$$

$$y_p[n]=-2\cdot 3^{-n}$$

$$y[n]=C_1+(C_2+nC_3)\frac{1}{2^n}-2\cdot 3^{-n}$$

$$\left.\begin{array}{l} y[0]=2=C_1+C_2-2 \\ y[1]=0=C_1+\frac{1}{2}C_2+\frac{1}{2}C_3-\frac{2}{3} \\ y[2]=2=C_1+\frac{1}{4}C_2+\frac{1}{2}C_3-\frac{2}{9} \end{array}\right\} C_1=92/9, C_2=-56/9 \text{ i } C_3=-116/9$$

7. zadatak (10 poena)

Koristeći osobinu razvoja u Furijeov red proizvoda dva signala (konvolucija) dobija se:

$$g[n]=x_1[n]x_2[n]\xleftrightarrow{FR} g_k=\sum_{q=\langle 4\rangle}\alpha_q\beta_{k-q}=\sum_{q=0}^{3}\alpha_q\beta_{k-q}=\alpha_0\beta_k+\alpha_1\beta_{k-1}+\alpha_2\beta_{k-2}+\alpha_3\beta_{k-3}=6$$

Dakle $g_k=6$ za svako k.